Educational
&
Didactic Communication
2010

Vzdelávacia
a
didaktická komunikácia
2010

The curriculum should be organized around models, not topics, because models are basic units of coherently structured knowledge, from which one can make direct inferences about physical systems and comparisons with experimental data. Cognition is basically about making and manipulating mental models.
David Hestenes: MODELING THEORY for math and Science Education, Arizona State University, 2007, U.S.A.

If the data is presented textually, the amount of data which can be displayed is in range one hundred data items, but it is like a drop in the ocean when dealing with data sets containing millions of data items.
For data mining to be effective, it is important to include the human in the data exploration process and combine the flexibility, creativity, and general knowledge of the human with the enormous storage capacity and the computational power of today´s computers.
Daniel A. Keim: Information Visualization and Visual Data Mining,, IEEE Transactions on Visualization and Computer Graphics, 2002, U.S.A.

In a nightmare world, we would perceive the world around us being continuous and withour structure. However, our survival as a species has been possible because we have evolved the ability do “cut up“ that world mentally into chunks about which we can think and hence give meaning.
This process of chunking, a part of all cognition, is modeling and the products of the mental actions that taken place are models.
John K.Gilbert: Visualization: An Emergent Field of Practice and Enquiry, Springer Science+Business Media, 2008, U.S.A.

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Educational & Didactic Communication 2010

Vzdelávacia a didaktická komunikácia 2010

Summary

The monograph Educational & Didactic Communication 2010 is a follow-up to earlier monographs which have presented the results of math, natural sciences, statistics and arts education in a field of didactic and communication of science based on the Brockmeyer´s communicative conception of physics and natural sciences education theory.

The communicative conception was described by the educational communication of physics/science (Brockmeyer, Kotásek) and by the curricular process (Průcha, Záškodný). The both conceptions were in 2008 integrated (Tarábek, Záškodný) and in 2009 implemented into data mining process (Záškodný, Procházka).

Following the communicative conception of natural sciences, statistics and arts education the model of educational data mining was developed in relation to phases and transformations of the educational communication and the curricular process.

For example, the mathematical data mining tools were used in an applied field of statistics education, in the derivation of financial option greeks on the basis of Black-Scholes model.

The monograph Educational & Didactic Communication 2010 is the result, by conceptually way elaborated by P.Tarábek and P.Záškodný, of the contributions from international internet conference "Curricular Process and Educational Data Mining" which has been carried out in Bratislava, Slovak Republic (June-September 2010). The conference promoters – Publishing House Didaktis, Curriculum Studies Research Group.

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Content

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Data Mining Tools in Statistics Education

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Abstract:

An imperative of data mining and a need of cooperation of the human with today´s computers are emphasized by D.A.Keim (see Zaskodny, Pavlat, 2009-2010a, Keim, 2002):

"The progress made in hardware technology allows today´s computer systems to store very large amounts of data. Researchers from the University of Berkeley estimate that every year 1 Exabyte (= 1 Million Terabyte) of data are generated, of which a large portion is available in digital form. This means that in the next three years more data will be generated than in all of human history before".

"If the data is presented textually, the amount of data which can be displayed is in range one hundred data items, but this is like a drop in the ocean when dealing with data sets containing millions of data items".

"For data mining to be effective, it is important to include the human in the data exploration process and combine the flexibility, creativity, and general knowledge of the human with the enormous storage capacity and the computational power of today´s computers."

The main principle of paper: Data Mining in Statistics Education (DMSTE) as Problem Solving
The main goal of paper: Delimitation of Complex Tool and Partial Tool of DMSTE
The procedure of paper: Data Preprocessing in Statistics Education
Data Processing in Statistics Education
Complex Tool of DMSTE – Curricular Process (CP-DMSTE)
Partial Tool of DMSTE – Analytical Synthetic Modelling (ASM-DMSTE)
Application of CP-DMSTE and ASM-DMSTE

The results of paper:

1. *Educational Communication of Statistics as Result of Data Preprocessing*
2. *Educational Communication of Statistics as Five Transformations* T1-T5 *of Knowledge from Statistics to Mind of Educant*
3. *Curricular Process of Statistics as Result of Data Processing*
4. *Curricular Process of Statistics as Structuralization, Algorithm Development and Formalization of Educational Communication of Statistics*
5. *Curricular Process as Succession of Five Transformations* T1-T5 *of Curriculum Variant Forms*
6. *Curriculum Variant Forms as Forms of Education Content Existence*
7. *Formalization of Curriculum Variant Form (Four of Universal Structural Elements: Sense and Interpretation, Set of Objectives, Conceptual Knowledge System, Factor of Following Transformation*)
8. *Variant Forms of Curriculum* – Conceptual Curriculum *(Communicable Scientific System of Statistics),* Intended Curriculum *(Educational System of Statistics),* Projected Curriculum *(Instructional Project of Statistics and Its Textbook),* Implemented Curriculum-1 *(Preparedness of Educator to Education),* Implemented Curriculum-2 *(Results of Education in Mind of Educant),* Attained Curriculum *(Applicable Results of Education)*
9. *Curricular Process as* CP-DMSTE *(Structuralization, Algorithm Development and Formalization of Five Transformations Succession* T1-T5*)*
10. *Analytical Synthetic Modelling as* ASM-DMSTE *(Modelling Inputs and Outputs of Transformations* T1-T5*)*
11. *Analytical Synthetic Models as Results of Problems Solving (Real or Mediated Problems)*
12. *Application of* CP-DMSTE *and* ASM-DMSTE *(Visualia of Conceptual Curriculum in Area of Statistics with Concrete Basic Statistical Set, Need of Visualiae of All Curriculum Variant Forms as Application of* CP-DMSTE*)*

Key Words:

Data Mining, Statistics Education, Statistics, Statistics with Concrete Basic Statistical Set, Data Mining Tool, Complex Tool, Partial Tool, Educational Communication, Curricular Process, Transformation of Knowledge, Variant Form of Curriculum, Visualia (Result of Visualization), Visualiae (Results of Visualization)

1. Data Mining (see also Supplement 1)

Data Mining – analytical synthetic way of extraction of hidden and potencially useful information from large data files (continuum data-information-knowledge, knowledge discovery)
Data Mining Techniques – the system functions of structure of formerly hidden relations and patterns (e.g. classification, association, clustering, prediction)
Data Mining Tool – a concrete procedure how to reach the intended system functions
Complex Tool – a resolution of complex problem of relevant science branch
Partial Tool – a resolution of partial problem of relevant science branch (e.g. analytical synthetic modeling, needful mathematical or statistical procedures)
Result of Data Mining – a result of data mining tool application
Representation of Data Mining Result – a description of this what is expressed
Visualization of Data Mining Result – optical retrieval of data mining result
Data Mining Cycle – Data Definition, Data Gathering, Data Preprocessing, Data Processing, Discovering Knowledge or Patterns, Representation and Visualization of Results

See P.Tarabek, P.Zaskodny, V.Pavlat, P.Prochazka, V.Novak, J.Skrabankova (2009-2010, 2009-2010abcde and quoted sources). **Quoted sources** in 2009-2010abcde:
E.g. American Library Association, M.C.Borba, E.M.Villarreal, G.M.Bowen, W-M Roth, C.Brunk, J.Kelly, R.Kohavi, Mineset, B.V.Carolan, G.Natriello, N.Delavari, M.R.Beikzadeh, S.Phon-Amnuaisuk, U-D Ehlers, J.M.Pawlowski, U.M.Fayyad, G.Piatelsky-Shapiro, P.Smyth, J.Fox, D.Gabel, J.K.Gilbert, O.de Jong, R.Justi, D.F.Treagust, J.H.Van Driel, M.Reiner, M.Nakhleh, W.Hämäläinen, T.H.Laine, E.Sutinen, M.Hesse, A.H.Johnstone, M.J.Kearns, U.V.Vazivani, D.A.Keim, R.Kwan, R.Fox, FT Chan, P.Tsang, Le Jun, J.Luan, J.Manak, National research Council-NRC, R.Newburgh, I.Nonaka, H.Takeuchi, C.J.Petroselli, E.F.Redish, D.Reisberg, C.Romero, S.Ventura, N.Rubenking, R.E.Scherr, M.Sabella, D.A.Simovici, C.Djeraba, V.Spousta, L.Talavera, E.Gaudioso, E.R.Tufte, J.Tuminaro, R.Vilalta, C.Giraud-Carrier, P.Brazdil, C.Soares, D.M.Wolpert.

2. Data Preprocessing in Statistics Education (see also Supplement 2)

Result of Data Preprocessing – Educational Communication of Statistics as a succession of transformations of education content forms (taken over from physics education):
- **The transformation T1** is transformation of scientific system of statistics to communicable scientific system of statistics (the first form of education content existence),
- **The transformation T2** is transformation of communicable scientific system of statistics to educational system of statistics (the second form of education content existence),
- **The transformation T3** is transformation of educational system of statistics to both instructional project of statistics and preparedness of educator to education (the third and fourth forms of education content existence),
- **The transformation T4** is transformation of both instructional project of statistics and preparedness of educator to results of education (the fifth form of education content existence),
- **The transformation T5** is transformation of results of statistics education to applicable results of statistics education (the sixth form of education content existence)

See J.Brockmeyer (1982), P.Zaskodny a kol. (2004, 2007), P.Tarabek, P.Zaskodny (2001, 2007-2008abc, 2008-2009, 2009-2010), P.Zaskodny (2001, 2006, 2009).

3. Data Processing in Statistics Education (see also Supplement 3)

Result of Data Processing – Curricular Process of Statistics as a succession of transformations of algorithmized and formalized education content forms (taken over from physics education):

i. The form of education content existence - "variant form of curriculum"

ii. The curriculum - "education content" (see Prucha, 2005)

iii. The variant forms of curriculum have got the universal structure (four structural elements - sense and interpretation, set of objectives, conceptual knowledge system, factor of following transformation)

iv. The variant forms of curriculum were selected on the basis of fusion of Anglo-American curricular tradition and European didactic tradition

v. The curricular process is defined as the succession of transformations T1-T5 **of curriculum variant forms:**

"**conceptual curriculum**" (output of T1, the first variant form of curriculum) - the communicable scientific system

"**intended curriculum**" (output of T2, the second variant form of curriculum) - the educational system of statistics

"**projected curriculum**" (output of T3, the third variant form of curriculum) - the instructional project of statistics

"**implemented curriculum-1**" (output of T3, the fourth variant form of curriculum) - the preparedness of educator to education

"**implemented curriculum-2**" (output of T4, the fifth variant form of curriculum) – the results of education

"**attained curriculum**" (output of T5, the sixth variant form of curriculum) - applicable results of education

See P.Prochazka, P.Zaskodny (2009-2010c). **Quoted sources** in 2009-2010c:
E.g. A.V.Kelly, M.K.Smith, W.Doyle, M.Pasch, A.M.Sochor, V.V.Krajevskij, I.J.Lerner, J.McVittie, K.Carter, G.M.Blenkin, L.Stenhouse, E.Newman, G.Ingram, F.Bobitt, R.W.Tyler, H.Taba, C.Cornblet, S.Grundy, D.Lawton, P.Gordon, M.Certon, M.Gayle, G.J.Posner.

4. Complex and Partial Tool of DMSTE – CP-DMSTE, ASM-DMSTE

Complex tool of DMSTE is given by curricular process of statistics (CP-DMSTE). CP-DMSTE delimits the correct education content via succession of transformations T1-T5.

Partial tool of DMSTE is given by analytical synthetic modelling (ASM-DMSTE). ASM-DMSTE describes the mediated or real problem solving within the inputs and outputs of individual transformations T1-T5. In this paper, the description of ASM-DMSTE is realized by means of both visualia Vis.1 and Legend to Vis.1.

Legend to Vis.1

$\boldsymbol{a}$ (Identified Complex Problem) – Investigated area of reality, investigated phenomenon

$\boldsymbol{B}_{\mathbf{k}}$ (Analysis) – Analytical segmentation of complex problem to partial problems

$\mathbf{b}_{\mathbf{k}}$ (Partial problems PP-k) – Result of analysis: essential attributes and features of investigated phenomenon

$\boldsymbol{C}_{\mathbf{k}}$ (Abstraction) – Delimitation of partial problems essences by abstraction with goal to acquire the partial solutions

$\boldsymbol{c}_{\mathbf{k}}$ (Partial solutions PS-k) – Result of abstraction: partial concepts, partial pieces of knowledge, various relations, etc.

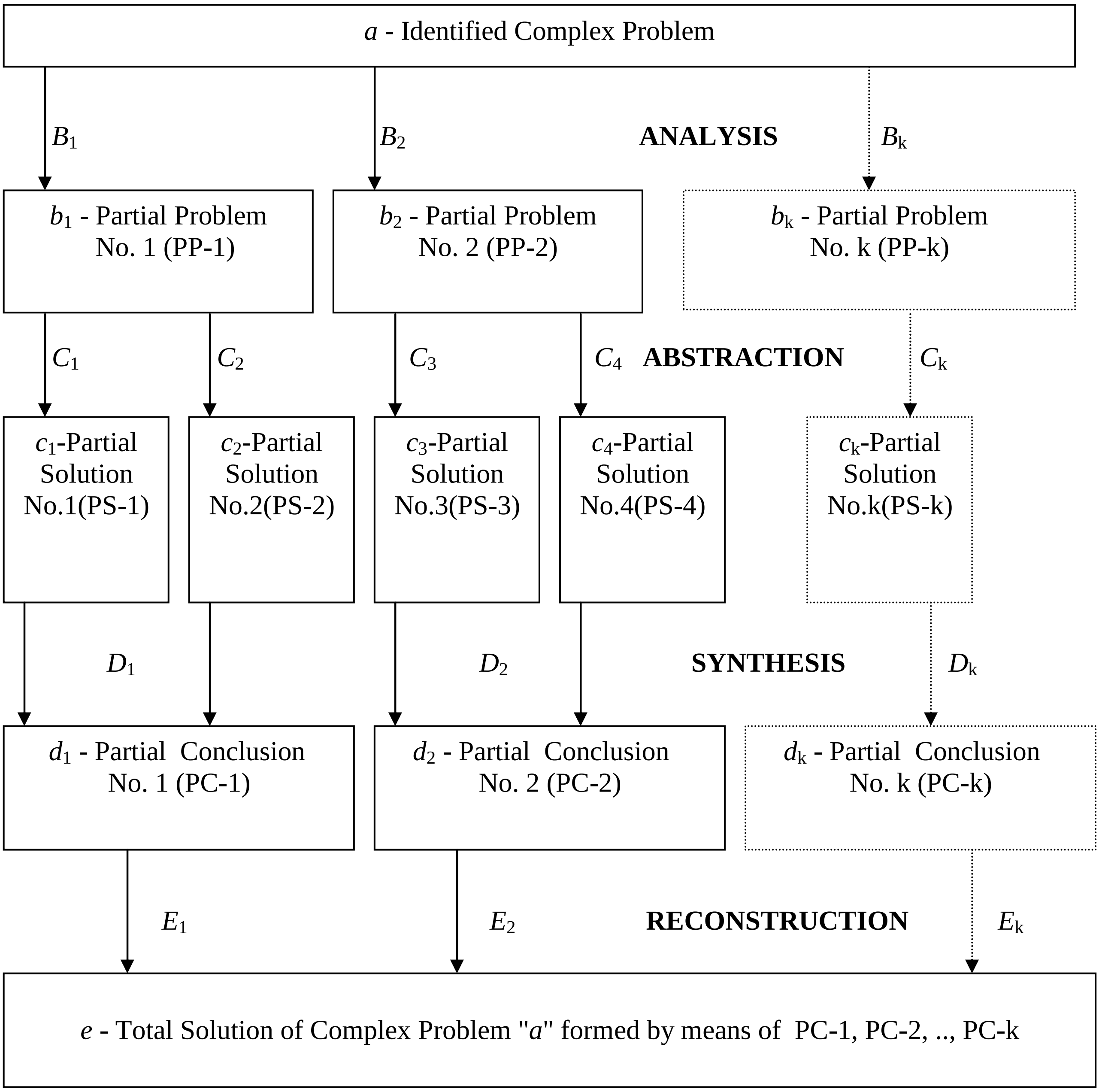
$\boldsymbol{D}_{\mathbf{k}}$ (Synthesis) – Synthetic finding dependences among results of abstraction

$\boldsymbol{d}_{\mathbf{k}}$ (Partial conclusions PC-k) – Result of synthesis: principle, law, dependence, continuity

$\boldsymbol{E}_{\mathbf{k}}$ (Intellectual reconstruction) – Intellectual reconstruction of investigated phenomenon / investigated area of reality

$\boldsymbol{e}$ (Total solution of complex problem "*a*") – Result of intellectual reconstruction: analytical synthetic structure of final knowledge (conceptual knowledge system)

Vis.1 General Analytical Synthetic Model of Problem Solving

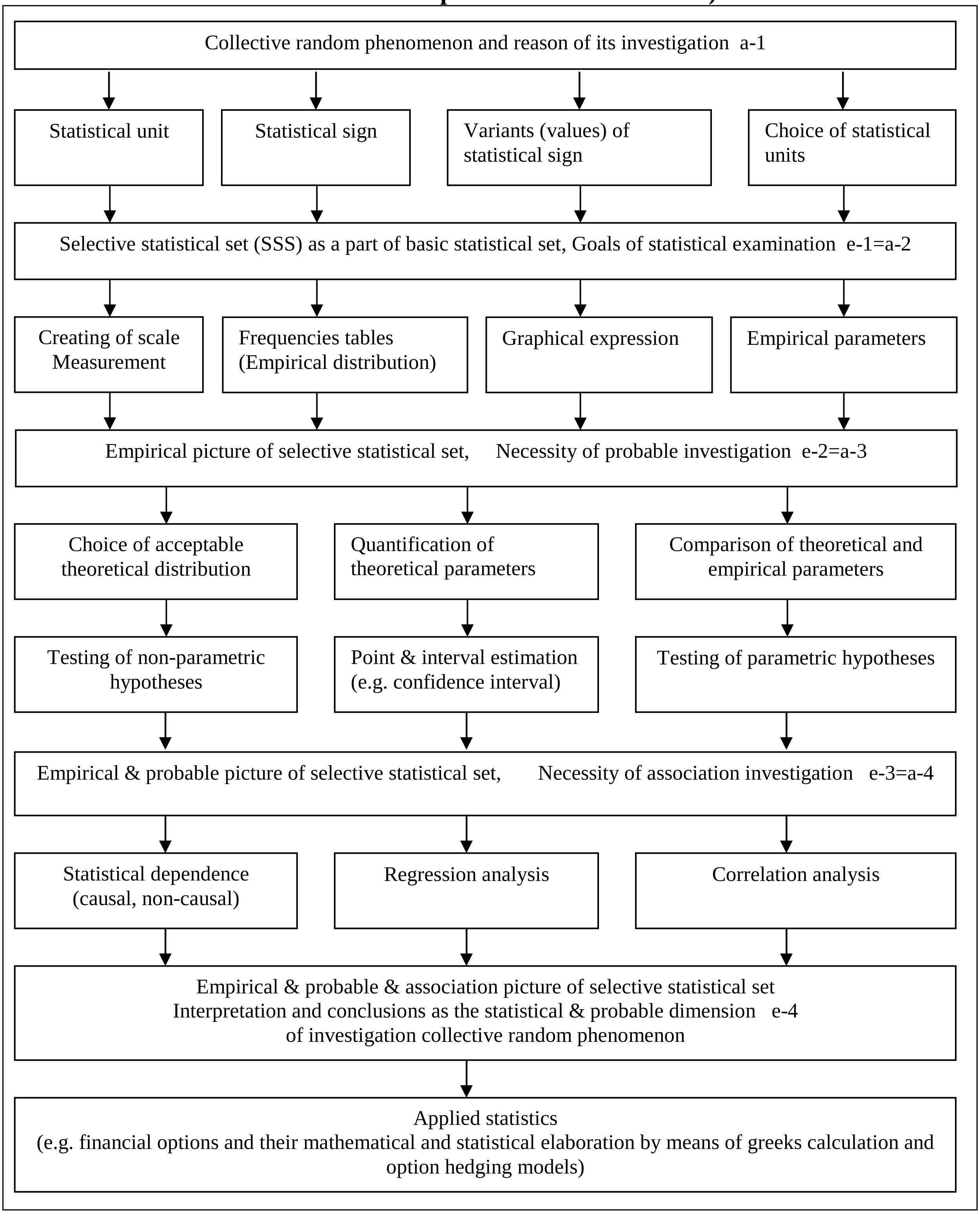


5. Application of Partial Tool ASM-DMSTE

The application of ASM-DMSTE is the visualia Vis.2 from the area of statistics education. The visualia Vis.2 is analytical synthetic model of statistics with concrete basic statistical set. This visualia constitutes a part of statistics conceptual curriculum as a part of communicable scientific system of statistics (a part of output of transformation T1).

The visualized result Vis.2 of data mining in statistics education constitutes the paramorphic model and hypertextual representation, represents the external conceptual knowledge systems as external representation of general social experience. The visualized result also represents the concrete type of data file – the representation of statistics with concrete basic statistical set.

Vis.2: An analytical synthetic model of statistics formed by four partial models a1-e1, a2-e2, a3-e3, a4-e4 (a part of conceptual curriculum of statistics – a part of communicable scientific system of statistics – output of transformation T1)



Remark: The visualization of conceptual curriculum of physics (classical, quantum and relativistic statistical and non-statistical physics) – **see English book** P.Zaskodny (2006): Survey of Principles of Theoretical Physics. The visualization of curricular process of physics – **see Czech book** P.Zaskodny (2009): Curricular Process of Physics

LEGEND to whole visualia Vis.2

a-1 → e-1 , a-2 → e-2 , a-3 → e-3 , a-4 → e-4

One – Sample Analysis, Two / Multiple – Sample Analysis

LEGEND to partial models of visualia Vis.2

a-1 → e-1

Formulation of statistical examination

a-2 → e-2

Relative & Cumulative Frequencies	(Empirical distribution)
Plotting functions: e.g. Plot Frequency Polygon	(Graphical expression)
Average-Means, Variance-Standard Deviation, Obliqueness, Pointedness	(Empirical parameters)

a-3 → e-3

Theoretical Distribution (partial survey in alphabetical order):
Bernoulli, Beta, Binomial, Chi-square, Discrete Uniform, Erlang, Exponential, F, Gamma, Geometric, Lognormal, Negative binomial, Normal, Poisson, Student´s, Triangular, Trinomial, Uniform, Weibull

Testing of Non-parametric Hypotheses (Hypothesis test for H0 – receive or reject H0):
e.g. computed Wilcoxon´s test, Kolmogorov-Smirnov test, Chi-square test
e.g. at alpha = 0,05

Point & Interval Estimation:
e.g. confidence interval for Mean, confidence interval for Standard Deviation

Testing of Parametric Hypotheses (Hypothesis test for H0 – receive or reject H0):
e.g. computed u-statistic, t-statistic, F-statistic, Chi-square statistic, Cochran´s test, Barlett´s test, Hartley´s test
e.g. at alpha = 0,05

a-4 → e-4

Statistical dependence:
e.g. confidence interval for difference in Means (Equal variances, Unequal variances)
e.g. confidence interval for Ratio of Variances

Regression analysis:
simple – multiple, linear – non-linear

Correlation analysis:
e.g. Rank correlation coefficient, Pearson´s correlation coefficoent

6. Conclusion

Modelling as a partial tool of data mining – quotation acoording to J.K.Gilbert (2008)**:**

"In a nightmare world, we would perceive the world around us being continuous and without structure. However, our survival as a species has been possible because we have evolved the ability do "cut up" that world mentally into chunks about which we can think and hence give meaning to".

"This process of chunking, a part of all cognition, is modelling and the products of the mental actions that have taken place are models. Science, being concerned with the provision of explanations about the natural world, places an especial reliance on the generation and testing of models".

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České Budějovice: University of South Bohemia, Czech Republic, http://sites.google.com/site/csrggroup/

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***Structural Textbook and Its Creation*.**
Bratislava, Slovak Republic: Didaktis, ISBN 80-85456-76-1

ii. Záškodný,P. (2001)
Statistical Dimension of Scientific Research.
KONTAKT, 2, 5, 2001 ISSN 1212-4117

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***Educational and Didactic Communication* 2007, *Vol*.1 – *Theory*.**
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***Educational and Didactic Communication* 2007, *Vol*.2 – *Methods*.**
Bratislava, Slovak Republic: Didaktis, www.didaktis.sk, ISBN 987-80-89160-56-3

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***Educational and Didactic Communication* 2007, *Vol*.3 – *Applications*.**
Bratislava, Slovak Republic: Didaktis, www.didaktis.sk, ISBN 987-80-89160-56-3

vi. Tarábek,P., Záškodný,P. (2008-2009)
***Educational and Didactic Communication* 2008.**
Bratislava, Slovak Republic: Didaktis, www.didaktis.sk, ISBN 978-80-89160-62-4

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Bratislava, Slovak Republic: Didaktis, www.didaktis.sk, ISBN 978-80-89160-69-3

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Základy zdravotnické statistiky.
České Budějovice, Czech Republic: South Bohemia University ISBN 80-7040-663-1

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***Survey of Principles of Theoretical Physics* (*with Application to Radiology*)**
(in English). Lucerne, Switzerland, Ostrava, Czech Republic: Avenira, Algoritmus, ISBN 80-902491-9-1

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(in Czech). Lucerne, Switzerland, Ostrava, Czech Republic: Avenira, Algoritmus, ISBN 978-80-902491-0-3

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Data Mining Tools in Science Education (in: vii.)

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Data Mining – A Brief Recherche (in: vii.)

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Collective Scheme of Both Educational Communication and Curricular Process (in: vii.)

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Modelling and Visualization of Problem Solving (in: vii.)

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Representation of Results of Data Mining (in: vii.)

Supplement 1
DATA MINING AND ITS TOOLS

(Retransfer – see References of Supplement 1)

S1.1. Quotations from Sources

S1.1.1. Definitions of Data Mining

J.Luan (2002)

Definition of Data Mining
a) Data Mining is the process of discovering meaningful new correlations, patterns, and trends by sifting through large amounts of data stored in repositories and by using pattern recognition technologies as well as statistical and mathematical techniques
b) The notion of Data Mining for higher education: Data Mining is a process of uncovering hidden trends and patterns that lend them to predicative modeling using a combination of explicit knowledge base, sophisticated analytical skills and academic domain knowledge

N.Rubenking (2001)

Definition of Data Mining
Data Mining is the process of automatically extracting useful information and relationships from immense quantities of data. In its purest form, Data Mining doesn´t involve looking for specific information. Rather than starting from a question or a hypothesis, Data Mining simply finds patterns that are already present in the data.

R.Kohavi (2000)

Definition of Data Mining as Knowledge Discovery
Data Mining (or Knowledge Discovery) is the process of identifying new patterns and insights in data

Interpretation of Data Mining
As the volume of data collected and stored in databases grows, there is a growing need to provide data summarization, identify important patterns and trends, and act upon findings

Le Jun (2008)

Definition of Data Mining as New Technology
Data Mining is extraction of hidden predictive information from large database. Data Mining is a powerful new technology with great potential to help an scientific area focus on the most important information in its data

N.Delavari, M.R.Beikzadeh, S.Phon-Amnuaisuk (2005)

Definition of Data Mining
Searched knowledge (meaningful knowledge, previously unknown and potentially useful information discovered) is hidden among the raw educational data set and it is extractable through Data Mining

R.Kwan, R.Fox, FT Chan, P.Tsang (2008), ***Le Jun*** (2008)

Data, Information, Knowledge
Data, Information, Knowledge are different terms, which differentiate in means and values.

a) Data is a collection of facts and quantitative measures, which exists outside of any context from which conclusions can be drawn.
b) Information is data that people interpret and place in meaningful context, highlighting patterns, causes of relationships in data.
c) Knowledge is the understanding human development as reaction to and use of information, either individually or as an organization.

Data-Information-Knowledge Continuum
a) Data, information and knowledge are separated but linked concepts which can form a data-information-knowledge continuum.
b) Data becomes information when people place it in context through interpretation that might seek to highlighting.
c) Knowledge can be described as a belief that is justified through discussion, experience and perhaps action. It can be shared with others by exchanging information in appropriate contexts.

S1.1.2. Data Mining and Problem Solving

L.Talavera, E.Gaudioso (2002)

Data Mining as Analysis Problem
In this paper we propose to shape the analysis problem as a data mining.

J.Tuminaro, E.F.Redish (2005), ***E.F.Redish*** (2005)

Problem solving
Problem solving and the use of math in physics courses
Student Use of Math in the Context of Physics Problem Solving: A Cognitive Model

M.C.Borba, E.M.Villarreal (2005)

Problem solving
Problem solving as context
Problem solving as skill
Problem solving as art

Process of modeling, process of problem solving
The process of modeling or model building is a part of the process of problem solving

Steps of problem solving process (process of problem solving as entailing several steps)
The starting point is a real problematic situation
The first step is to create a real model, making simplifications, idealizations, establishing conditions and assumptions, but respecting original situation
In the second step, the real model is mathematized, to get a mathematical model
The third step implies the selection of suitable mathematical methods and working within mathematics in order to get some mathematical results
In the fourth step, these results are interpreted for and translated into the real situation

S1.1.3. Forms of Data Mining, Data Mining System, Goals of Data Mining, Scope of Data Mining

R.Kohavi (2000)

Forms of Data Mining (Structured mining etc.)
Structured mining, Text mining, Information retrieval

W.Hämäläinen, T.H.Laine, E.Sutinen (2003)

Data Mining system, educational system
Data Mining system in educational system: the educational system should be served by Data Mining system to monitor, intervene in, and counsel the teaching-studying-learning process

R.Kohavi (2000)

Goals of Data Mining
Data Mining serves two goals:
-Insight: Identified patterns and trends are comprehensible
-Prediction: A model is built that predicts (scores) based on input data. Prediction as classification (discrete variable) or as regression (continuous variable)

Scope of Data Mining
The majority of research in DM has concentrated on building the best models for prediction.
A learning algorithm is given the training set and produces a model that can map new unseen data into the prediction.

S1.1.4. Results of Data Mining, Applications of Data Minings, Interdisciplinarity of Data Mining

R.Kohavi (2000), ***D.M.Wolpert*** (1994), ***M.J.Kearns, U.V.Vazivani*** (1994)

Some theoretical results in Data Mining
- No free lunch (All concepts are equally likely, then learning is impossible)
- Consistency (non-parametric models - target concept given enough data, parametric models as linear regression are known to be of limited power) - enough data = consistency
- PAC learning (probably approximately correct learning) is a concept introduced to provide guarantees about learning
- Bias-Variance decomposition

U.M.Fayyad, G.Piatelsky-Shapiro, P.Smyth (1996)

Interdisciplinarity of Data Mining
Data Mining, sometimes referred to as knowledge Discovery, is at the intersection of multiple research area, including machine learning, statistics, pattern recognition, databases and visualization

J.Luan (2002)

Potential applications of Data Mining
"There are several ways to examine the potential applications of Data Mining
a) One is to start with the functions of the algorithms to reason what can be utilized for
b) Another is to examine the attributes of a specific area where data are rich, but mining activities are scare
c) And another is to examine the different functions of a specific area to identify the needs that can translate themselves into Data Mining project"

Notes: a) - See Curricular Process as Data Mining Algorithm
b) - See Curriculum: Theory and Practice as scientific area in which mining activities are scare
c) - Some of the most likely places where data miners (educational researchers who wear this hat) may initiate Data Mining projects are: Variant Forms of Curriculum

S1.1.5. Data Mining techniques

.
N.Delavari, M.R.Beikzadeh, S.Phon-Amnuaisuk (2005)

Data Mining techniques
"DM techniques can be used to extract unknown pattern from the set of data and discover useful knowledge. It results in extracting greater value from the raw data set, and making use of strategic resources efficiently and effectively."

J.Luan (2001)

Data Mining techniques as Data Mining functions
"Prediction, clustering, classification, association"

Le Jun (2008)

Data Mining techniques – application of Data Mining tools
"Application of DM tools: To solve the task of prediction, classification, explicit modeling and clustering. The application can help understand learners´ learning behaviors"

C.Romero, S.Ventura (2006)

Data Mining techniques in educational systems
"After preprocessing the available data in each case, **Data Mining techniques** can be applied in educational systems – statistics and visualization, clustering, classification and outlier detection, association rule mining and pattern mining, text mining"

J.Luan (2002)

Clustering and prediction – the most striking aspects of Data Mining techniques
- "The clustering aspect of Data Mining offers comprehensive characteristics analysis of investigated area"
- "The predicting function estimates the likelihood for a variety of outcomes"

B.V.Carolan, G.Natriello (2001)

Clustering
"Data-Mining Resources to identify structural attributes of educational research community-e.g. clustering as collaboration of physicists and biologists"

D.A.Simovici, C.Djeraba (2008)

Clustering, Taxonomy of clustering
a) "**Clustering** is the process of grouping together objects that are similar. The groups formed by clustering are referred to as **clusters**."
b) "**Clustering can be regarded as a special type of classification**, where the clusters serve as classes of objects"
c) "It is widely used data mining activity with **multiple applications** in a variety of scientific activities from biology and astronomy to economics and sociology"
d) "**Taxonomy of clustering** (we follow here the taxonomy of clustering)
- Exclusive or nonexclusive: Clustering may be exclusive or may not be exclusive. It is exclusive, where an exclusive clustering technique yields clusters that are disjoint. It is nonexclusive, where a nonexclusive technique produces overlapping clusters.

- **Intrinsic or extrinsic:** Clustering may be intrinsic or extrinsic. Intrinsic - based only on dissimilarities between the objects to be clustered. Extrinsic - which objects should be clustered together and which should not, such information is provided by an external source.
- **Hierarchical or partitional:** Clustering may be hierarchical or partitional. Hierarchical - in hierachical clustering algorithms, a sequence of partitions) is constructed. Partitional - partitional clusterings creates a partition of the set of objects whose blocks are the clusters such that objects in a cluster are more similar to each other than to objects that belong to different clusters"

S1.1.6. Data Mining tools

C.Brunk, J.Kelly, R.Kohavi (1997)

Data Mining tool
""Mineset" is a Data Mining tool that integrates Data Mining and visualization very tightly. Models built can viewed and interacted with."

C.Romero, S.Ventura (2006)

Data Mining tools
"Data Mining tools provide mining algorithms, filtering and visualization techniques. The examples of Data Mining tool:
- Tool name: Mining tool, Authors: Zaïane and Luo (2001), Mining task: Association and patterns
- Tool name: Multistar, Authors: Silva and Vieiva (2002), Mining task: Association and classification
- Tool name: Synergo/ColAT, Authors: Avouris et al (2005), Mining task: Visualization"

D.A.Simovici, C.Djeraba (2008)

Mathematical tools for Data Mining
a) "This book was born from experience of the authors as researches and educators, which suggests that **many students of Data Mining are handicapped** in their research by the lack of formal, systematic education in its mathematics. The book is intended as a reference for the working data miner."
b) "In our opinion, **three areas of math are vital for DM:**
- set theory, including partially ordered sets and combinatorics,
- linear algebra, with its many applications in principal component analysis and neural networks,
- and probability theory, which plays a foundational role in statistics, machine learning and DM"

S1.1.7. Modelling, Model

J.K.Gilbert, M.Reiner, M.Nakhleh (2008), ***J.K.Gilbert*** (2008), ***J.K.Gilbert, R.Justi*** (2002)

Definition of Modelling, Model
"We have evolved the ability do "cut up" that world mentally **into chunks** about which we can think and hence **give meaning to. This process of chunking** (Data Mining clustering), a part of all cognition, **is modelling** and the products of the mental actions that have taken place **are models**"

Significance of Modelling, Model
"Modelling as an element in scientific methodology and **models** at the outcome of modelling are both important aspects of the conduct of science and hence of science education"

"Categorization of models
a) Historical models (**Curriculum models**) - learning specific consensus (the P-N junction model of transistor). Curriculum models can be used to provide an acceptable explanation of

a wide range of phenomena and specific facts, that´s why, it is useful way of reducing, by chunking, the ever-growing factual load of **science curriculum**
b) **New qualitative models** - developed by following the sequence of learning: To revise an established model, To construct a model de novo (to reconstruct an established model)
c) **New quantitative models** - developed by following the sequence of learning: quantitative version of a useable qualitative model of phenomenon
d) **Progress in the scientific enquiry** is indicated by the value of particular **combination of qualitative and quantitative models** in making successful predictions about it properties"

C.M.Borba, E.M.Villarreal (2005)

Definition of modeling
"Modeling can be understood as a pedagogical approach that emphasizes students´ choice of a problem to be investigated in the classroom. Students, therefore, play an active role in curriculum development instead of being just the recipients of tasks designed by others."

"Problem solving
- problem solving as context
- problem solving as skill
- problem solving as art"

Process of modeling, process of problem solving
"The process of modeling or model building is a part of the process of problem solving."

"Steps of problem solving process
Process of problem solving as entailing several steps:
a) **The starting point** is a real problematic situation
b) **The first step** is to create a real model, making simplifications, idealizations, establishing conditions and assumptions, but respecting original situation
c) **In the second step**, the real model is mathematized, to get a mathematical model
d) **The third step** implies the selection of suitable mathematical methods and working within mathematics in order to get some mathematical results
e) **In the fourth step**, these results are interpreted for and translated into the real situation"

J.K.Gilbert, O.de Jong, R.Justi, D.F.Treagust, J.H.van Driel (2002)

"Model as a major learning and teaching tool
Models are one of the main products of science, modelling is an element in scientific methodology, (and) models are a major learning and teaching tool in science education"

"Model of Modelling Framework
1. **Decide on purpose - Select source for model** and Have experience - Produce mental model
2. Produce mental model - Express in mode(s) of representation
3. Express in mode(s) of representation - Conduct thought experiments
4a. Conduct thought experiments (pass) - Design and perform empirical tests
4b. Conduct thought experiments (fail) - Reject mental model (Modify mental model) and **back to Select source for model** (negative result)
5a. Design and perform empirical tests (pass) - Fulfill purpose and Consider scope and limitations of model and **back to Decide on purpose** (positive result)
5b. Design and perform empirical tests (fail) - Reject mental model (Modify mental model) and **back to Select source for model** (negative result)"

R.Justi, J.K.Gilbert (2002)

"Role of chemistry textbooks in the teaching and learning of models and modelling

This role may be discussed from two main angles:
- the way that chemical models are introduced in textbooks
(**note:** projected curriculum, a learning model)
- and the teaching models that they present
(**note:** Implemented curriculum-1, a teaching model)"

"Teaching model, Learning model, Analogies
A teaching model is a representation produced with the specific aim of helping students to understand some aspect of content. Assuming the abstract nature of chemical knowledge, they (**learning models**) are used very frequently in chemical textbooks mainly **in the form of overt analogies**, as drawings and as diagrams (specifically to "the atom", "chemical bonding" and "chemical equilibrium")"

"Some future research directions
a) How can teachers´**pedagogical content knowledge** about models and modelling be improved?
b) The role of models and modelling in the **development of chemical knowledge**?
c) How can it be made evident to teachers that the introduction of model-based teaching and learning approach can be way to shift the emphasis in chemical education **from transmission of existing knowledge to a more contemporary perspective in which students will really understand the nature of chemistry** and be able to deal critically with chemistry-related situations?"

S1.1.8. Representation (Creativity)

J.K.Gilbert, M.Reiner, M.Nakhleh (2008), ***J.K.Gilbert*** (2008)

"Levels of Representation
The **"Representation in Science Education"** is concerned with challenges that students face in understanding the three **"levels"** at which models can be **represented** - "macro", "sub-micro", "symbolic" - and the relationships between them."

A.H.Johnstone (1993), ***D.Gabel*** (1999)

"Representations as distinct representational levels
a) The models produced by science are expressed in three distinct representational levels
b) The macroscopic level - this consists of what is seen in that which is studied
c) The sub-microscopic level - this consists of representations of those entities that are inferred to underlie the macroscopic level, giving rise to the properties that it displays - molecules and ions are used to explain the properties of pure solutions, of radiotherapy)
d) The symbolic level (this consists of any qualitative abstractions used to represent each item at the sub-microscopic level - chemical equations, mathematical equations)"

J.K.Gilbert (2008), ***M.Hesse*** (1966), ***G.M.Bowen, W.-M.Roth*** (2005))

"The ontological categorization of representations
a) **Two approaches** to the ontological categorization of representations are put forward, one based **on the purpose** which the representation is intended to serve, the other **on the dimensionality -** 1D,2D,3D - of the representation.
b) **The purpose for which a Model is Produced**
- All models are produced by the use analogy. The target (which is the subject of the model) is depicted by a partial comparison with a source. The classification is binary: The target and the source are the same things (they are homomorphs - an aeroplane, a virus), They are not (they are paramorphs - paramorphs are used to model process rather than objects)
c) **The dimensionality of the Representation**

The idea that modelling involves the progressive reduction of the experienced world to a set of abstract signs can be set out in terms of dimensions are follows:
- Macro level - Perception of the world-as-experienced - 3D, 2D
- Sub-micro level - Gestures, concrete representations (structural representations) - 3D
  - Photographs, virtual representations, diagrams, graphs, data arrays - 2D
- Symbolic level - Symbols and equations - 1D"

E.R.Tufte (1983), ***J.K.Gilbert*** (2008), ***D.Reisberg*** (1997)

"External and internal representations, Series of internal representations and creativity
a) Visualization is concerned with **External Representation,** the systematic and focused public display of information in the form of pictures, diagrams, tables, and the like
b) Visualization is also concerned with **Internal Representation,** the mental production, storage and use of an image that often (but not always) is the result of external representation
c) **External and internal representations are linked** in that their perception uses similar mental processes
d) Visualization is thus concerned with **the formation of an internal representation from an external representation. An internal representation** must be capable of mental use in the making of predictions about the behaviour of a phenomenon under specific conditions
e) It is entirely possible that once **a series of internal representations have been visualized**, that they are amalgamated/recombined to form a novel internal representation that is capable of external representation **- this is creativity**"

S1.1.9. Visualization

J.K.Gilbert, M.Reiner, M.Nakhleh (2008), ***J.K.Gilbert*** (2008)

Definition of Visualization
"The making of meaning for any such representation is "visualization". Visualization is central the production of representations of these models (curriculum models, qualitative and quantitative models and their combinations)."

J.K.Gilbert (2008)

Visualization and Internal Representation
"Visualization is also concerned with **Internal Representation,** the mental production, storage and use of an image that often (but not always) is the result of external representation."

R.Kohavi (2000)

"Essence of Visualization - Data Summarization
As the volume of data collected and stored in databases grows, there is a growing need to provide data summarization (**e.g. through visualization**), identify important patterns and trends, and act upon findings."

C.Brunk, J.Kelly, R.Kohavi (1997)

"Serviceability of Visualization
One way to did users in understanding the models is to visualize them."

D.A.Keim (2002)

"Serviceability of Visualization

a) Information Visualization techniques may help to solve the problem
b) Data Mining will use Information Visualization technology for an improved data analysis"

Application of Visualization
"Application of Visualization is Visual Data Exploration"

"Benefits of Visual Data Exploration
- University of Berkeley - every year 1 Exabyte of data (10^{18} bytes, Gigabyte = 10^{9} bytes)
- Finding the valuable information hidden in them, however, is a difficult task
- The data presented textually - The range of some one hundred data items can be displayed (a drop in the ocean)
- **The basic idea of visual data exploration** is to present the data in some visual form, allowing the human to get insight into the data, draw conclusions, and directly interact with the data (to combine the flexibility, creativity and general knowledge of the human with the enormous storage capacity and the computational power of today´s computers)
- **The visual data exploration process** can be seen a hypothesis generative process (coming up with new hypotheses and the verification of the hypotheses can be done via visual data exploration)
- **The main advantages of visual data exploration:** Visual data exploration can easily deal with inhomogenous and noisy data, **visual data exploration** is intuitive and requires no understanding of mathematical and statistical algorithms, **visual data exploration techniques** are indispensable in conjuction with automatic exploration techniques
- **Visual data exploration paradigm:** overview first, zoom and filter, details-on-demand"

S1.1.10. Metavisualization

N.R.C. (2006)

"Metavisualization - spatial thinking
The associated visualization which can be called "spatial thinking""

J.K.Gilbert, M.Reiner, M.Nakhleh (2008), ***J.K.Gilbert*** (2008),

"Metavisualization - learning from representations
It is of such importance in science and hence in science education that the acquisition of fluency in visualization is highly desirable and may be called "metavisual capability" or **"metavisualization".** A fluent performance in visualization has been described as requiring metavisualization and involving the ability to acquire, monitor, integrate, and extend learning from representations. **Metavisualization - learning from representations."**

"Criteria for Metavisualisation
Four criteria are suggested for attainment of metavisual status. The person concerned must be able to:
a) demonstrate an understanding of the "convention of representation" for all the modes and sub-modes of 3D,2D,1D representations (what they can and cannot represent)
b) demonstrate a capacity to translate a given model between the modes and sub-modes in which it can be depicted
c) demonstrate the capacity to be able to construct a representation within any mode and sub-mode of dimensionality for a given purpose
d) demonstrate the ability to solve novel problems using a model-based approach"

"Developing the Skills of Metavisualization
level 1 - representation as depiction
level 2 - early symbolic skills
level 3 - syntactic use of formal representations
level 4 - semantic use of formal representations
level 5 - reflective, rhetorical use of representations"

S1.1.11. Visual DM techniques

D.A.Keim (2002)

"Classification of Visual Data Mining Techniques (abstraction criterium)
- Techniques as x-y plots, line plots, and histogram, but they are limited to relatively and low-dimensional data sets
- Novel information visualization techniques allowing visualization of multidimensional data without inherent 2D or 3D semantics."

D.A.Keim (2002)

"Classification of Visual DM Techniques based on three criteria a), b), c)

a) The data to be visualized (one or two- dimensional data, multidimensional data, text and hypertext, hierarchies and graphs, algorithms and software):
Dimensionality of date set = the number of variables of data set.
Text and hypertext = in the age of the world wide web one important data type is text and hypertext
Hierarchies and graphs = data records often have some relationship to other pieces of information, i.e. a graph consists of set objects, called nodes, and connections between these objects, called edges.
Algorithms and software = the goal of V is to support software development by helping to understand algorithms, e.g. by showing the flow of information in a program, to enhance the understading of written code, e.g. by representing the structure of thousands of source code lines as graphs

b) The visualization techniques (Standard 2D/3D displays, Geometrically-transformed displays, Icon-based displays, Dense pixel displays, Stacked displays-treemaps, dimensional stacking)
Geometrically-transformed displays = these techniques aim at finding "interesting" transformations of multidimensional data sets. The class of geometric display techniques includes also the well-known Parallel Coordinate Technique (PCT). The PCT maps the k-dimensional space onto the two display dimensions by using k equidistant axes which are parallel to one of display axes
Icon-based displays = the idea is to map the attribute values of a multidimensional data item to the features of an icon

c) **The interaction (IT) and distortion (DT) techniques used** (interactive projection, interactive filtering, interactive zooming, interactive distortion, interactive linking and brushing)
Interaction techniques allow the data analyst to directly interact with visualizations and dynamically change the visualizations according to exploration objectives
Distortion techniques help in the data exploration process by providing means for focusing on details while preserving an overview of the data
***Interactive filtering, Interactive zooming* -** in exploring large data sets it is important to interactively partition the data into segments and focus on interesting subsets. This can be done by a direct selection of the desired subset (BROWSING) or by a specification of properties of the desired subset (QUERYING)."

S1.1.12. Educational Data Mining

C.Romero, S.Ventura (2006)

Educational Data Mining
a) "Currently there is an increasing interest in DM and educational systems (well known learning content management systems, adaptive and intelligent web-based educational systems), **making educational Data Mining as a new growing research community** (from 1995 to 2005)."

b) "After preprocessing the available data in each case, **Data Mining techniques** can be applied in educational systems – statistics and visualization, clustering, classification and outlier detection, association rule mining and pattern mining, text mining"
c) "**Data Mining oriented towards students** – to show recommendations and students to use, interact, participate and communicate within educational systems (e.g. traditional classrooms)"
d) "**Data Mining oriented towards educators (and academic responsible-administrators)** – to show discovered knowledge and educators (administrators) to design, plan, build and maintenance within educational systems (e.g. traditional classrooms)"
e**) "Data Mining tools** provide mining algorithms, filtering and visualization techniques. The examples of Data Mining tool:
- Tool name: Mining tool, Authors: Zaïane and Luo (2001), Mining task: Association and patterns
- Tool name: Multistar, Authors: Silva and Vieiva (2002), Mining task: Association and classification
- Tool name: Synergo/ColAT, Authors: Avouris et al (2005), Mining task: Visualization"

f) "**Future researches lines in educational Data Mining**
- Mining tools more easy to use by educators or not expert users in Data Mining
- Standardization of data and methods (preprocessing, discovering, postprocessing)
- Integration with the e-learning system
- Specific Data Mining techniques"

W.Hämäläinen, T.H.Laine, E.Sutinen (2003)

Data Mining system, educational system
"Data Mining system in educational system: the educational system should be served by Data Mining system to monitor, intervene in, and counsel the teaching-studying-learning process"

R.E.Scherr, M.Sabella, E.F.Redish (2007)

Curriculum development
"Conceptual knowledge is only one aspect of good knowledge structure: how and when knowledge is activated and used are also important."

Representation of knowledge structure
"The nodes represent knowledge. The lines represent relations between different nodes."

R.Newburgh (2008)

"Linear and lateral (structural) thought process (in physics)
Why do we lose physics students?
a) There is a wide spectrum in thought process. Of the two major types one is **linear** (i.e. sequential) and the other **lateral** (i.e. seeking horizontal connections).
b) Those who developed physics - from Galileo to Newton to Einstein to Heisenberg - were almost exclusively **linear thinkers**. Paradigm for linear thought is **Eucledian thinking**, Eucledian logic (many physicists chose physics for their career as a result of their exposure to geometry - a consequence of this is that textbooks are usually written in a Eucledian format). **The sense of discovery is lost.** Many students do not recognize that the Eucledian format is not a valid description how we do physics. Their way of approaching problems is different but just as valid. Too many physics teachers refuse to recognize the limitations of this approach (thereby causing would-be students who do not think in a Eucledian fashion to leave).
c) **The format of our textbooks is Eucledian.** Newton´s laws, Hamilton-Jacobi theory, and Maxwell´s equations are often presented as quasi-axioms in advanced texts. The laboratories become fixed exercises in which the student must confirm some principle already established. He knows the answer before he does the experiment.
d) Now I yield to no one in my **admiration for Euclid**. He has been an inspiration to many of us. We understand his genius but also see his limitations. **Unfortunately there are many who do not follow his way of thinking.**

e) By presenting alternate approaches to students (**specifically uses of lateral thinking**), false starts that must be corrected, and **lessons that are discoveries not memorization**, we can retain more students in physics.
f) **We should remember that lateral thinking is essential to the formation of analogies**, an activity that one cannot describe as Euclidean. Doing science without analogies seems to me an impossibility."

J.K.Gilbert, O.de Jong, R.Justi, D.F.Treagust, J.H.van Driel (2002), ***R.Justi, J.K.Gilbert*** (2002)

Model as a major learning and teaching tool
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"This role may be discussed from two main angles:
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a) "How can teachers´**pedagogical content knowledge** about models and modelling be improved?"
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J.K.Gilbert, O.de Jong, R.Justi, D.F.Treagust, J.H.van Driel (2002), ***J.H.van Driel*** (2002)

"Curriculum for Chemical Eduaction
a) The central question is concerns **the design of curricula for chemical education** (**note:** curricular process) which make chemistry interesting and relevant for various groups of learners (professional chemists, general educational purposes-it is useful for all citizens in the future)
b) In recent decades, **curricula have been changed**, **on the one hand for general educational purposes,** this has led to context-based approaches to teaching chemistry, **on the other hand for professional chemists** specific chemistry courses have been developed in the context of vocational training, aimed at developing the specific chemical competencies that are needed for various professions.
c) Finally, chemistry is nowadays also presented in informal ways, for instance, in science centres and through chemistry "shows"."

U-D.Ehlers, J.M.Pawlowski (2006)

"Quality and Standardization in E-learning
- Quality development: Methods and approaches
Methods, models, concepts and approaches for the development, management and assurance of quality in e-learning are introduced
- E-learning standards
The main goal of e-learning standards is to provide solutions to enable and ensure interoperability and stability of systems, components and objects."

R.Kwan, R.Fox, FT Chan, P.Tsang (2008), ***Le Jun*** (2008)

Knowledge management, Data Mining
"We set up a few objects and value propositions of the initiative which was set up to improve teaching and learning, to enhance the quality of curriculum, and to extent learning support. We apply Data Mining tools to discover behavioral characteristics. A few strategies for knowledge management in the curriculum development in distance education will be discussed."

Le Jun (2008), ***I.Nonaka, H.Takeuchi*** (1995), ***I.Nonaka, H.Takeuchi*** (2005)

Types of knowledge, Interaction of types
"Many knowledge management experts agree that there are **two general types of knowledge:**

a) Tacit knowledge is linked to personal perspective intuition, emotion, belief, experience and value. It is intangible, not easy to articulate, and difficult to share with others.
b) Explicit knowledge has a tangible dimension that can be more easily captured, codified and communicated

Based on ***I.Nonaka, H.Takeuchi*** **these two versions of knowledge can interact** when the "knowledge conversion" occurs:
- socialization: from tacit to tacit
- externalization: from tacit to explicit
- combination: from explicit to explicit
- internalization: from explicit to tacit"

Le Jun (2008), ***I.Nonaka, H.Takeuchi*** (2005)

"Research methods for knowledge management
a) **Data Mining techniques**
b) **Web text mining** is discovery knowledge from based non-structural text (text representation, feature extraction, text categorization, text clustering, text summarization, semantic analysis, and information extraction)
c) **Learning theory**
Learning theories are classified into four paradigms: behavioral theory, cognitive theory, constructive theory, social learning theory.
We emphasize: Learning is continuous process that was indistinguishable from ongoing work practice - by discovering the problems, recognizing their types, and by solving problems in routine work and learning. Learners can continuously refine their cognitive, information, social and learning competencies.
d) **Knowledge management**
Knowledge sharing and application of the SECI model (see ***I.Nonaka, H.Takeuchi***)"

S1.1.13. Metadata Mining Process

R.Vilalta, C.Giraud-Carrier, P.Brazdil, C.Soares (2004)

Meta-learning – Support Data Mining
„Current data mining tools are characterized by a plethora of algorithms but a lack of guidelines to select the right method according to the nature of the problem under analysis. Producing such guidelines is a primary goal by the field of meta-learning; the research objective is to understand the interaction between the mechanism of learning and the concrete contexts in which that mechanism is applicable. The field of meta-learning has seen continuous growth in the past years with interesting new developments in the construction of practical model-selection assistants, task-adaptive learners, and a solid conceptual framework. In this paper, we give an overview of different techniques

necessary to build meta-learning systems. We begin by describing an idealized meta-learning architecture comprising a variety of relevant component techniques. We then look at how each technique has been studied and implemented by previous research. In addition, we show how metalearning has already been identified as an important component in real-world applications.“

J.Fox (2007)

Definition Metadata Mining process
“Since metadata is just another type of data, applying data mining to metadata is technically straightforward. **XML -** *eXtensible Markup Language*”

American Library Association (1999)

“Definition of Metadata
a) As for most people the difference between data and information is merely a philosophical one of no relevance in practical use, other definitions are:
Metadata is information about data.
Metadata is information about information.
Metadata contains information about that data or other data
b) There are more sophisticated definitions, such as:
Metadata is structured, encoded data that describe characteristics of information-bearing entities to aid in the identification, discovery, assessment, and management of the described entities.”

S1.2. Brief Summary of S1.1.

Data Mining – an analytical synthetic way of extraction of hidden and potencially useful information from the large data files (continuum data-information-knowledge, knowledge discovery)
Data Mining Techniques – system functions of the structure of formerly hidden relations and patterns (e.g. classification, association, clustering, prediction)
Data Mining Tool – a concrete procedure how to reach the intended system functions
Complex Tool – a resolution of the complex problem of relevant science branch
Partial Tool – a resolution of the partial problem of relevant science branch
Result of Data Mining – a result of the data mining tool application
Representation of Data Mining Result – a description of this what is expressed
Visualization of Data Mining Result – an optical retrieval of the data mining result

S1.3. Data Mining Cycle

S1.3.1. Quotations from Sources

U.M.Fayyad, G.Piatelsky-Shapiro, P.Smyth (1996)

“Cycle of Data mining
Data Mining can be viewed as a cycle that consists of several steps:
- Identify a problem where analyzing data can provide value
- Collect the data
- Preprocess the data obtain a clean, mineable table
- Build a model that summarizes patterns of interest in a particular representational form
- Interpret/Evaluate the model
- Deploy the results incorporating the model into another system for further action.”

J.Luan (2002)

“Steps for Data Mining preparation (algorithm, building, visualization)
a) Investigate the possibility of overlaying Data Mining algorithms directly on a data warehouse
b) Select a solid querying tool to build Data Mining files. These files closely resemble multidimensional cubes
c) Data Visualization and Validation. This means both examining frequency counts as well as generating scatter plots, histograms, and other graphics, including clustering models
d) Mine your data”

Le Jun (2008)

“Main processes of Data Mining
- The main processes include data definition, data gathering, preprocessing, data processing and discovering knowledge or patterns (Data Mining techniques can be implemented rapidly on existing software and hardware)
- Application of Data Mining tools: To solve the task of prediction, classification, explicit modeling and clustering. The application can help understand learners´learning behaviors.”

S1.3.2. Brief Summary of S1.3.1.

Data Mining Cycle:

- Data Definition, Data Gathering
- Data Preprocessing, Data Processing
- Data Mining Techniques and Data Mining Tools,
- Discovering Knowledge or Patterns,
-Representation and Visualization of Data Mining Results,
- Application.

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ii. Záškodný,P., Pavlát,V. (2009-2010a)
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Data Mining – A Brief Summary (in: i.)

Supplement 2
DATA PREPROCESSING – EDUCATIONAL COMMUNICATION OF PHYSICS

(Retransfer – see References of Supplement 2)

S2.1. Data Definition, Data Gathering – Description of Educational Communication of Physics

The subject of physics education (the educational communication of physics within the communicative conception - J.Fenclova-Brockmeyerova, 1982) does not identify with the subject of educational science. It includes the whole knowledge of physics with its ties to the physics´ surroundings and also the education of physics in the society (J.Fenclova-Brockmeyerova, 1982, in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010).

The educational communication of physics is the subject of physics education in the communicative conception (this includes also methodic, integrative and application conception – P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010).

The physics education is a whole continuous process of forwarding and negotiation of results and methods of physics knowledge to the sense of individuals, who are not directly bounded with the knowledge creation. This process is leading to the transfer of physics knowledge to the sense of whole society (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010). This process is done by various participants with educational intention and includes not only the teaching and education in all levels of educational system, but also lifelong studies carried out institutionally and information transfer from physics science to society.

The physics knowledge piece undergoes several transformations during the educational communication (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010). Physics education has to follow the whole way of physics knowledge transfer and thus is passed through totally different areas of thinking, investigation and locution, which approximately come up to mentioned physics knowledge transformation. These are the basic problematic areas of physics didactics.

The basic problematic areas of physics education are: the scientific system of physics, the educational system of physics, instructional project – instructional process, the results of instruction and their evaluation. Also the teacher's preparation of physics and methodology of physics education belong to this group of problematic areas of physics education. The transformations of physics knowledge piece and inputs and outputs of each transformation define these basic problematic areas of physic education (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010).

Physics conceptual-knowledge systems undergo several forms of existence during the educational communication of physics and gain this forms of existence in transformations T1 to T5 (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010).

The survey of transformations could be described according to Brockmeyer, 1982 (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) and P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010, respectively following way:

1. The ascertainment of objective reality, usually made by measuring, an experiment, or mathematical deduction, must be defined in words, mathematically, or graphically in order to be publishable and thus conveyable to the widest circle of physicists, and at the same time filed into the scientific system of physics (J.Fenclová-Brockmeyerová, 1982, in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)

Transformation T1 (input → output) (P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)
Input T1: The scientific system of physics (the system of physical theories → Output T1: The scientific system of physic in the light of its communicability (the scientific system of physic from the point of view of its communication).

2. Such a piece of physics knowledge is transformed to the educational system of physics, with regard to the addressees and objectives, of which the system is to serve its purpose (J.Fenclová-Brockmeyerová, 1982, in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010).

Transformation T2 (input → output) (P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)
Input T2: Scientific system of physic from the point of view of its communication → Output T2: Educational system of physics and its content (for example, contents of instruction as a subject matter - the most precise and concise expression of the educational system of physics)

3. Another transformation occurs when trying to express the piece of knowledge (usually simplified) didactically in a particular product, such as textbook (J.Fenclová-Brockmeyerová, 1982, in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010).

Transformation T3 (input → output) (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)
Input T3: Educational system of physics and its content → Output T3: Instructional project of physics and its textbook (the textbook as an important part of instructional project of physics)

4. Another transformation occurs during the process of teaching and learning. This transformation can be multipled, the piece of knowledge can be passed on gradually and enriched in its content. The result is knowledge in the form of information (a piece of learning). (J.Fenclová-Brockmeyerová, 1982, in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)

Transformation T4 (input → output) (P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)
Input T4: Instructional project of physics and its textbook, learners´ initial information and experience (including common knowledge), preparedness of teacher → Output T4: Results of physics education (the transformation T4 is realized through the instruction of physics at school)..

5. Another transformation happens when incorporating the piece of knowledge into the education as a lasting value of man, in his consciousness and awareness. Another kind of transformation is the application of the physics knowledge in the real life or a production situation ((J.Fenclová-Brockmeyerová, 1982, in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)

Transformation T5 (input → output) (P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)
Input T5: The results of physics education (the outputs of physical schooling → Output T5: Applied results of physics education (permanent constituent of education and applied outputs of physical schooling).

S2.2. Brief Summary of Data Preprocessing

The physical dimension of physics education is evident from the description of the educational communication of physics according to J.Fenclová-Brockmeyerová (1982, p.26, in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010), and P.Tarabek and P. Zaskodny (2006, p.149, in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010). A piece of physics knowledge undergoes several distinctive transformations during the didactic communication:

Transformation of the piece of physics knowledge (J.Fenclova-Brockmeyerova, in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010): The physics education must follow all the way of passing of the physical knowledge and it goes through completely different areas of thinking, research and expression, which roughly correspond with the outlined transformations. These are the basic problematical areas of physics education

Transformation of the piece of physics knowledge (P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) Within the frame of the theory of „Didactic Communication“ is introduced a succession of **T1 to T5 Transformations** of a piece of physics knowledge from the scientific language to the language of methodology, which is comprehensible enough to both teachers and students.

S2.3. Need of Structuralization, Algorithm Development, and Formalization

The physical dimension of physics education is associated with the above described succession of the transformations of the pieces of physics knowledge. **Here come up three questions associated with these transformations (the need of a structuralization, an algorithm development, and a formalization):**

Question One: How to create, express and display the inputs and outputs of T1 to T5 Transformations? How to create, express and display the scientific system of physics education, so that it would be communicable? How to create, express and display the didactic system of physics education, its objectives and subject matter? How to create, express and display the elements of an educational project of physics, especially the textbook? How to convey the knowledge to the students, so that it would become their knowledge (learning, information), and how to find out about, display and evaluate achieved results of teaching physics at schools? How to assess, display and evaluate the applicability of acquired physics knowledge?

Question Two: How do the educational and physical dimension of physics education relate? Suppose that the educational and physical dimension of physics education are associated with the transformations of the piece of physics knowledge, then which educational constructs to use in order to describe particular transformations?

Question Three: Can we consider the first two questions as substantial also for the field education of scientific disciplines, eventually also for the subject education of entirely different field areas?

The common answer to all of the three questions (and also a fulfillment of the requirement of data structuralization, data algorithm development, and data formalization) is closely related to the variant forms of curriculum existence (**i. e. the theory and practice of curriculum**), and to the ways of the expressing and constructing of these types of curricula with appropriate educational constructs (**i. e. with appropriate modeling of the contents of education in its variant forms of existence**). The succession of the variant forms of curriculum can be called as **„curricular process“** (Manak, 2005, Zaskodny, 2007 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010), similarly for the name of the succession of T1 to T5 transformations of physics knowledge the common term of „educational communication of physics“ can be used.

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Supplement 3
DATA PROCESSING – CURRICULAR PROCESS OF PHYSICS

(Retransfer – see References of Supplement 3)

S3.1. Investigation of Curriculum Variant Forms in Publications

There are many definitions of curriculum. Some of them will be used, briefly discussed and only approximately applied to the results and course of T1 to T5 transformations. These applications do not aspire to becoming a comprehensive association and comparative – their goal is to show the link to the particular transformations of physics knowledge, and to try to name preliminarily this link with the help of an appropriate variant form of curriculum existence.

a) A.V.Kelly, 1999

Curriculum is all the learning which is planned and guided by the school, whether it is carried on in groups or individually, inside or outside the school. Curriculum has two basic features: learning is planned and guided, and the definition of curriculum is related to school education – **the results of T3 transformation, the course of T4 transformation**

b) M.K.Smith, 2000

In what follows we are going to look at four ways of enquiry into curriculum theory and practice:

Curriculum as a Body of Knowledge to be transmitted, Curriculum as a Syllabus to be transmitted. Such curriculum orientates itself to the canon of Western cultural heritage and is associated with systematic development of reasoning power and the communication of the canon. It should be transmitted or 'delivered' to students by the most effective methods (Blenkin, 1992 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) - **the results of T1 and T2 transformation.**

Curriculum as Product. We think of product as specific activities that are needed for life – Bobbitt, 1918, 1928 (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010). Curriculum as product focuses on the statement of changes to take place in the students. Curriculum as product emphasizes the environment of the educational purposes, and education associated with it the rationality and relative simplicity (Tyler, 1949 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010), and is very similar to technical or productive thinking (Taba, 1962). A feature (and perhaps an inconvenience, too) of this model is assumption, that behaviour is objectively and mechanistically measurable (Stenhouse, 1975, Cornbleth, 1990 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) **- the results of T4 and T5 transformation**.

Curriculum as Process emphasizes the interaction of teachers, students and knowledge, curriculum is what actually happens in the classroom and what people do to prepare and evaluate. The most significant is natural order of development in the student – curriculum is in harmony with the child's real interests, needs and learning patterns. A curriculum is rather like "a recipe in cookery". (Stenhouse, 1975 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010). These processes can enhance a person´s learning, if they work well, or, if they go wrong, inhibit a person's learning'. By this process the learning is offered, accepted and internalized (Newman, Ingram, 1989) **- the results of T3, and the course of T4 transformation**.

Curriculum as Praxis. Curriculum as praxis is, in many respects, a development of the process model. This model brings to the centre the emphasis on collective human well-being and the emancipation of the human spirit. School is a major, perhaps the principal force for social change and social justice. The curriculum is not simply a set of plans to be implemented, but rather is constituted through an active process in which planning, acting and evaluating are all reciprocally related and integrated into the process. (Grundy, 1987 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) **- the course of T4 transformation, the results of T4 transformation.**

c) J.McVittie, 2007 a, 2007 b

Three different kinds of curriculum will be discussed – transmissional or technical (learning exists outside the man and is objectively determined and researchable), next transactional, or practical (learning originates in interaction with other people and the world, learning is conveyed and constructed socially and personally), and transformational or emancipatory (students accept the world view of the dominant group, and this „false consciousness“ needs to be transformed in order to reach the full potential of the student).

Various educational models are associated with particular types of curriculum (conceptual – the model of information processing, behavioral – the model of direct teaching, cooperative – the model of social interaction, empirical – personal model), and learning theory (e.g. behavioral, constructivist).

For instance, with the transactional type of curriculum are linked conceptual, cooperative and empirical models of education. With these educational models go teaching methods. The empirical educational model is accompanied by the teaching methods such as excursions (field trips), guided discovery, predict-observe-explain, playing-thinking-sciencing), concept formation, concept attainment, independent research, and particular information jig saw. With cooperative educational model are associated all teaching methods associated with empirical model and also role-play and simulation. With conceptual teaching model go all teaching methods associated with cooperative and empirical educational model, and also the teaching method of concept web and concept map.

The word „curriculum“ comes from Latin and its common meaning is the course of studies. A teacher will say „curriculum“ meaning a curriculum handbook. Parents will sometimes say „curriculum“ meaning the content of the course.

For the purpose of curriculum studies at the University of Saskatchewan the word curriculum will cover all that represents „favourable conditions“ – „aspices“ – for school functioning. As part of this curriculum there can be included equipment (part of hidden curriculum), the contents of curriculum handbooks, the way we teach/manage/evaluate acts and behavior, school sponsors´ activities, (part of ineffective or „nul curriculum“) etc. The curricula in according to Saskatchewan are transactional, fixed in the constructivist theory of learning and conceptually oriented. Conceptual orientation **- the results of T1 and T2 transformation**

Within the framework of core curriculum studies seven basic areas of curriculum studies are required – linguistics, mathematics, science, social studies, medical education, humanities, and physical education. The curriculum studies will be aimed at required curricular documentation that arises from SaskLearning, as well as the ways of teaching and evaluating - **the results of T3 transformation, and the course and results of T4 transformation**

d) J.Průcha, 2005

<u>Definition of Curriculum:</u>

* Curriculum in the narrow sense of the word means the programme of teaching (Lawton, Gordon, 1993 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)

* Curriculum in the broad sense of the word means all learning happening in schools or other institutions, both planned and not planned. (Lawton, Gordon, 1993 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)

* Recently curriculum has been defined as a digest of the culture of the society, and the curriculum is created in the process of cultural analysis. (Lawton, Gordon, 1993 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)

* Curriculum is the content of entire experience, that students gain at school and during activities related to school, its planning and evaluation (Průcha, Walterová, Mareš, 2001 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010).

* From the point of view of curriculum research and the very essence of education process curriculum is understood as the content of education, or curricular content – every educational process has two subjects and its own content (the content as curriculum is a subject matter of learning and also teaching in school environment)

* Curriculum should be understood as a sum of all that is taught at school and what children learn (Doyle, 1992 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)

* Curricular research makes it possible to define curriculum towards building the teaching material with such qualities which would respect logical and semantic structures which would make learning easier. A.M.Sochor (1974) (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) enriched the curriculum theory by working on the problems of communicability of the subject matter, and in terms of measurable semantic networks at that. (Sochor, 1974 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)

Content Pedagogy Theory

* This practical construction line in the curriculum theory represents the research of ways of transforming the contents of human learning in particular sciences (Doyle, 1992a - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010). This is connected with Doyle´s definition of curriculum.
* Content Pedagogy Theory (postulated by Doyle) researches into particular procedures of choice, structure and organization of what is supposed to be the content of school education. Authors of curriculum try to generate the most useful forms of learning plans, educational standards and texts in order to use them immediately in school education practice (see also Sochor, 1974 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010). This is also dealt with in more detail in J.Průcha works (1987) (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)
* Walter Doyle is Professor in College of Education at the University of Arizona. His other works also include, for example, W.Doyle, K.Carter (2003) (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)

Five Curriculum Concepts (Certon, Gayle, 1991):
* The concepts related to the structure of learning (the subject matter as a sum of knowledge items of particular sciences):
T1 and T2 Transformation, Conceptual and Intended Curriculums
* The concepts of the development of cognitive processes (thinking is more important than the facts)
T4 Transformation, Implemented Curriculum
* The concepts related to the technology of teaching (learning focused on the method of imparting)
T3 Transformation, Projected Curriculum
* The concepts of child´s self-fulfillment (giving the learner space to discover the world through thein own activities, arising from their own interests):
T4 Transformation, Implemented Curriculum
* Concepts of rectification of the society (solving society´s abuses through education)
T5 Transformation, Attained Curriculum
According to J.Průcha (2005, p 239) (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010): „Members of particular opposing parties can hardly come to terms with the advocates of a different approach, which often leads to sectarianism and rivalry among authors of various curriculums"

Forms of Curriculums:
* Conceptual Form (Concept of what is to be the contents of education):
T1 and T2 Transformation, Conceptual and Intended Curriculums
* Projected Form (particularly planned projects of the contents of education:
T3 Transformation, Projected Curriculum
* The Form of realization (the contents of education presented to the subjects of education):
T3 Transformation, Projected Curriculum and Implemented Curriculum-1
***** Result Form (the contents of education perceived by the subjects of education):
T4 Transformation, Implemented Curriculum-2
Effect Form (the contents of education functioning on the side of the subjects of education):
T5 Transformation, Attained Curriculum

The approach to curriculum as a variant phenomenon was worked out in theory as early as in 1980s (Průcha, 1987 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010). According to J.Průcha (2005) (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) it became the base of empirical analyses, concern of which, apart from curricular theory, was the following of practical aims: To carry through the view, that "planned contents of education is not identical with the „realized contents of education" („there is difference between what the authors of the curriculum plan, and what pupils are able to learn")

The notions of Intended Curriculum, Implemented Curriculum and Attained Curriculum are analyzed as 3 different levels in the following way (Straková, Tomášek, Palečková, 1996 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010):

Intended Curriculum – intended aims and content of education explicitly defined in curricular documents (curriculum, textbooks). Three categories of content can be distinguished: the educational content itself, its operational level (the activities of students and teachers e.g. when handling appropriate types of learning tasks), and the level of prospects (planned changes of learners´ attitudes, intersts and motivations)

Implemented Curriculum - the learning matter actually delivered to the students within the frame of particular school education (that is by a particular teacher in a particular school)

Attained Curriculum - the learning matter actually attained by students. One important form of the educational content is its form modified by the students based on their own, including extra-curricular experience and interests.

The mechanisms of the transfer between the forms of curriculum are described only partially. In the least way the transformations of project form of curriculum into the realisation level of curriculum (**see Transformation as a transfer from Project Curriculum and Implemented Curriculum-1 to Implemented Curriculum-2**). According to V.V.Krajevskij and I.J.Lerner (1983) (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) important changes occur during this transformation. In the project form there are (Krajevskij, Lerner, 1983 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) two forms of educational content: Invariant content of education (determined by curricular documents – **Project Curriculum as a partial result of T3 Transformation**), and variant content of education (content interpreted by a particular teacher in a particular class - **Implemented Curriculum 1 as a teacher´s preparation for class and as another result of T3 Transformation**)

<u>Levels of curriculum construction:</u>
* Curriculum in class and at school (Doyle, 1992b - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010), curriculum „comes to life" only in class, when it is realised by a teacher in the process of teaching. Its part is also Projected Curriculum on the school level.
* Curriculum in educational programmes (Projected Curriculum is usually defined in curricular documents such as school curricula and educational programmes, curriculum, methodology handbooks for teachers, exam requirements, etc.), educational programme represents a kind of curricular document on the national level, and in international terminology it is associated with national curriculum (as projected curriculum guaranteed by the state), or framework curriculum.

e) J.Svoboda, R.Kolářová (2006)
<u>Elements of Projected National Curriculum</u>

An educational programme as Projected National Curriculum (for the educational system of the country or for a certain type of school and as a complex system of managing **the contents of school education** on the national level) usually consists of the following elements (Průcha, 2005 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010):
- education concept
- aims of particular education
- syllabus
- subject matter (its themes) in particular subjects or integrated subject fields
- target standards as requirements of what learners are expected to learn in particular years or levels of the school

- evaluation tools for finding out about accomplishing target standards
- implementation plan, with the help of which the educational program will be implemented into educational practice

<u>Projected Curriculum on National Level in the Czech Republic (Czech National Curriculum)</u>

The need to develop new educational programmes for basic and middle schools in the Czech Republic emerged from the government document called „Concept of education and development of the educational system in the Czech Republic“ The principles of the formation of Czech national curriculum as project curriculum on the national level were compiled in the „National programme of the development of education in the Czech Republic (White Book, 2002 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) and brought forward in the Act of Pre-school, Basic, Secondary, Tertiary Professional and Other Education (since 2005). Curricular doccuments of the Czech national curriculum are formed, apart from the „National Programme of the Development of Education“, by so called „Framework Educational Programme“ and „Framework Syllabus“ for pre-school, basic, gymnasium, secondary professional and other education (**T3 Transformation, Project Curriculum on national level).**

The Framework Educational Programme for basic and secondary education divides its contents into 9 educational areas

Language and Language Communication
Mathematics and its Application
Man and the World
Man and Society
Man and Nature
Art and Culture
Man and Heath
Man and the World of Work
Information and Communication Technology

These are made up of closely-related educational fileds (field methodologies as arts that deal with particular educational fields). While forming project curricula on school level the schools can split these educational fields into subjects (subject methodologies as arts that deal with particular subjects). On the basic education level the subject of physics assumes to be included in years starting with the sixth year of Basic school and year Prima on the lower level of eight-year Gymnasium.

The Framework Syllabus for basic and secondary education determines mandatory implementing of educational areas into the basic, secondary and secondary professional education, and allocates minimum time endowment, bound dispensable time endowment, (for the 2nd level of basic school), free dispensable time endowment, and total obligatory time endowment for a particular level of education. The Framework Syllabus assesses the duty to include cross-sectional themes, and encourages creation of integrated subjects.

For instance, the educational content of physics at basic school is, according to the Framework Educational Programme made up of such cross-sectional themes like Matters and Bodies, Movements of Bodies, Forces, Mechanical Characteristics of Liquids, Energy, Sound Processes, Electromagnetic and Light Processes, and the Universe. At Gymnasium, according to the Framework Educational Programme the cross-sectional themes are Physical Values and their Measuring, Mechanics, Molecular Physics and Thermics, Electromagnetism, Optics, Physics of Microworld, and Astrophysics.

<u>Projected Curriculum on School Level</u>

On the basis of the Framework Educational Programme school educational programmes are being created (recently for all basic schools and for several pilot Gymnasiums in the Czech Republic), these divide the educational kontent into particular subjects on the basis of the Framework Educational

Programme for a particular school level. (**T3 Transformation, Project Curriculum on School Level**). The educational content specified in this way is then structured by means of curricula for particular subjects in the following way:
- name of the subject
- characteristics of the subject
- educational content of the subject (outputs from the Framework Educational Programme to years, selection and processing of the subject matter into years, and in association with expected outputs, thematic areas of cross-sectional themes, inter-subject context.

f) T.Janík (2005)

The content of education (curriculum) is not only the knowledge planned for teaching (in order to become students´ knowledge), but also planned experience, skills, values, attitudes and interests, which are to be developed in students.

The pragmatic concept of curriculum in the USA - in 1950s the teaching of subjects interconnects with mother disciplines at the universities. Curriculums are created, which are contentually derived from the systematics and logics of scientific disciplines (**T1 and T2 Transformations, Conceptual and Intended Curriculum)** Voices have grown stronger which point at the need to choose the core curriculum

In the structure of scientific disciplines one can distinguish the substantive structure (concepts which create the content load) and the syntax structure (ways of findings, which lead from the data through their interpretation to a conclusion) (Schwab, 1964 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) – **the result of T1 Transformation, Conceptual Curriculum**

In the research work on pedagogy there appears the need for a new research paradigm – the category of the content of the education should be taken seriously enough into consideration (Shulman, 1987 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010). From there goes the research on methodological knowledge of the content targeted at inquiry into the professional knowledge of teachers. (Janík, 2004 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)

In recent years the metodology has come (under the influence of the constructivist theory of learning) to the search for domain specific corresponding with the particularities of the content of a specific subject (**T1 and T2 Transformation, Conceptual and Intended Curriculum**)

g) J.Mańák, 2005

Within the framework of curricular research we can distinguish
- Official Curriculum (recorded in school documentation) - **T3 Transformation, Projected Curriculum**
- Operational Curriculim, taught at schools - **T3 Transformation, Implemented Curriculum-1 as teacher´s preparation for instruction**
- Hidden Curriculum (which adds information, e.g. from student´s environment
- Nul Curriculum (regards subjects not taught at the school)
- Extra-Curriculum (expected experience goes beyond the frame of the subject) (Poster, 1992 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010).

The research also regards the following
- Curriculum as expected results (**T4 Transformation, Implemented Curriculum-2 as subject matter mastered by students)**
- Curriculum as structures of particular knowledge areas (**T1 and T2 Transformation, Conceptual and Intended Curriculum**)
- Influence upon human behaviour and stimuli to productive and creative thinking (**e.g. T5 Transformation, Attained Curriculum**)

The structure of educational aims has become the subject matter of theoretical analyses:
- the highest educational aims, sophisticated taxonomies for the cognitive area are available (Brunner, Bloom, Gagne), affective (Kathwohl) and psychomotoric (Harrow) – **T2 Transformation, Intended Curriculum**

- from the highest educational aims particular goals are derived as requirements of particular subjects **- T2 Transformation, Intended Curriculum**
- particular goals lead into specifying objectives, which are associated with teaching units, courses and tasks - **T3 Transformation, Projected Curriculum, Implemented Curriculum-1 as teacher´s preparation for instruction.**

The expertise and research of learning texts also predicate of the concept of curriculum (**T1 and T2 Transformation, Conceptual and Intended Curriculum**) and point out at:
- the choice of the subject matter and involvement of circumstances (**creating, expressing and displaying results of T2 Transformation, i.e Intended Curriculum**)

An important role in the process of curricular research is played by „Curricular process“ as an analysis of instruction and the process of „capturing“ the subject matter by the students (**T4 Transformation, from Project Curriculum and Implemented Curriculum-1 as teacher´s preparation for instruction to Implemented Curriculum-2 as the subject matter mastered by students**). This „Curricular Process“ takes its course in the most effective way as problem solving (**tip show to display results of T2 and T4 Transformations, i.e. Intended Curriculum and Implemented Curriculum-2. Displaying these results as models of intermediated problem solving based on the merging of the systemic and logic approaches**).

There also appear problems of curriculum associated with growing influence of postmodern concepts – those make the existing canon of traditional values and educational aims relative. (Alba, 2000 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010).

S3.2. Brief Summary of Data Processing - Curricular Dimension of Physics Education

The gradual development of the variant forms of curriculum is like a „process of capturing the subject matter by the students“ newly specified as **„Curricular process“** (Manak, 2005, Zaskodny, 2007 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010). Resulting from the research of theory and practice of curricular process it becomes evident that the succession of T1 to T5 of Transformations of pieces of physics knowledge (**educational communication of physics education**) can be associated with the succession of the 5 variant forms of curriculum (**curricular process of physics**) in the following way:

i. The form of education content existence **- “variant form of curriculum**”
ii. The curriculum - “**education content**” (see Prucha, 2005 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010)
iii. The variant forms of curriculum have got the universal structure (four structural elements - sense and interpretation, set of objectives, conceptual knowledge system, factor of following transformation)
iv. The variant forms of curriculum were selected on the basis of fusion of Anglo-American curricular tradition and European didactic tradition
v. The curricular process is defined as the succession of transformations T1-T5 **of curriculum variant forms:**

“**conceptual curriculum**” (output of T1, the first variant form of curriculum) - the communicable scientific system of physics (or other natural science)

“**intended curriculum**” (output of T2, the second variant form of curriculum) - the educational system of physics (or other natural science) "**projected curriculum**" (output of T3, the third variant form of curriculum) **-** the instructional project of physics (or other natural science)

"**implemented curriculum-1**" (output of T3, the fourth variant form of curriculum) **-** the preparedness of educator to education

"**implemented curriculum-2**" (output of T4, the fifth variant form of curriculum) – the results of education

"**attained curriculum**" (output of T5, the sixth variant form of curriculum) **-** applicable results of education

See P.Prochazka, P.Zaskodny (2009-2010c) (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010). **Quoted sources** in 2009-2010c:
E.g. A.V.Kelly, M.K.Smith, W.Doyle, M.Pasch, A.M.Sochor, V.V.Krajevskij, I.J.Lerner, J.McVittie, K.Carter, G.M.Blenkin, L.Stenhouse, E.Newman, G.Ingram, F.Bobitt, R.W.Tyler, H.Taba, C.Cornblet, S.Grundy, D.Lawton, P.Gordon, M.Certon, M.Gayle, G.J.Posner.

The above described association as **Curricular Dimension of Physics Education** simplifies and unites the terminology of physics education at basic and secondary schools, and also collegiate physics education – the transformation of the piece of physics knowledge can for all kinds of schools be described as a succession of consecutive variant forms of curriculum as curricular process of physics.

According to J.Průcha (2005, p. 239) (in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010), regarding the five concepts of curriculum (Certon, Gayle, 1991 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) „Members of particular opposing parties can hardly come to terms with the advocates of a different approach, which often leads to sectarianism and rivalry among authors of various curriculums".

We can assume that if the didactic communication of physics is taken as a subject of physics education (Fenclová-Brockmeyerová, 1982 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010), the above described antagonism among the members of particular opposing parties can, surprisingly easily, be to a certain extent eliminated. Particular concepts, particular variant forms of curriculum only express different transformations of the piece of physics knowledge. From this point of view the particular concepts of curriculum could cooperate very well. Besides, in the Czech Republic the approach to the curriculum as variant phenomenon was theoretically drawn up as early as in the 1980s. (Průcha,1987 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010). The discovery of „didactic communication of physics" (Fenclová-Brockmeyerová, 1982 - in P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) and its structural construction with the help of T1 to T5 Transformations (P.Tarabek, P.Zaskodny, 2007-2008a.b.c, 2008-2009, 2009-2010) can, through its interconnection and sequence of concepts and particular forms of curriculum, acknowledge the productivity of the approach to the curriculum as variant phenomenon.

At the same time there appears one important role of physics education as one of subject didactics – the physics education with its curricular dimension defines itself as a unique and independent scientific field.. The curricular dimension of physics education can also be the way to the structural approach to physics education.

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Partial Data Mining Tools in Applied Statistics – in Greeks and Option Hedging

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Abstract:

An imperative of data mining and a need of cooperation of the human with today´s computers are emphasized by D.A.Keim (Keim, 2002):

"The progress made in hardware technology allows today´s computer systems to store very large amounts of data. Researchers from the University of Berkeley estimate that every year 1 Exabyte (= 1 Million Terabyte) of data are generated, of which a large portion is available in digital form. This means that in the next three years more data will be generated than in all of human history before".

"If the data is presented textually, the amount of data which can be displayed is in range one hundred data items, but this is like a drop in the ocean when dealing with data sets containing millions of data items".

"For data mining to be effective, it is important to include the human in the data exploration process and combine the flexibility, creativity, and general knowledge of the human with the enormous storage capacity and the computational power of today´s computers."

Financial options are those derivative contracts in which the underlying assets are financial instruments such as stocks, bonds or an interest rate.

The Black-Scholes model traces the evolution of the option´s key underlying variables in continuous-time. **The Binomial and Trinomial models** trace the evolution of the option's key underlying variables in discrete-time.

In mathematical finance, **the Greeks** are the quantities representing the sensitivities of derivatives such as options to a change in underlying parameters on which the value function of an instrument or portfolio of financial instruments is dependent. The name is used because the most common of these sensitivities are often denoted by Greek letters.

The Greeks in the Black-Scholes model are relatively easy to calculate, a desirable property of financial models, and are very useful for derivatives traders, especially those who seek to hedge their portfolios from adverse changes in market conditions. For this reason, those Greeks which are particularly for Hedging Delta, Gamma and Vega are well-defined for measuring changes in Price, Time and Volatility.

The basic concepts about Data Mining in Statistics Education – see Záškodný,P., Tarábek,P. (2010-2011), ***Data Mining Tools in Statistics Education*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.). The complex data mining tool in statistics education is the Curricular Process of Statistics. The partial data mining tools in statistics education are the Analytical Synthetic Modelling, Statistical and Mathematical Data Mining Tools.

The main goal of presented paper – description of Statistical and Mathematical Data Mining Tools in Statistics Education.

The main principle of paper: Data Mining in Statistics Education (Greeks and Option Hedging)
The main goal of paper: Partial Tools of DMSTE – Statistical Tools, Mathematical Tools
The procedure of paper: Financial Options – Introduction, Statistical Base of Financial Option
Greeks and Option Hedging in Black-Scholes and Binomial Models
Statistical Data Mining Tools – Normal, Binomial and Trinomial Distributions
Mathematical Data Mining Tools

The results of paper:

1. *Description of Financial Options*
2. *Description of Statistical Base of Financial Option*
3. *Greeks in Black-Scholes Model*
4. *Greeks in Binomial Model*
5. *Option Hedging in Black-Scholes Model*
6. *Option Hedging in Binomial Model*
7. *Statistical Data Mining Tools in Applied Statistics – in Greeks and Option Hedging*
8. *Mathematical Data Mining Tools in Applied Statistics – in Greeks and Option Hedging*

Key Words:

Data Mining, Statistics Education, Partial Data Mining Tool, Financial Options, Greeks, Option Hedging, Black-Scholes Model, Binomial Model, Trinomial Model, Statistical Data Mining Tools, Mathematical Data Mining Tools

1. Financial Options - Introduction

(quoted according to www.economywatch.com)

Financial options are those derivative contracts in which the underlying assets are financial instruments such as stocks, bonds or an interest rate. The options on financial instruments provide a buyer with the right to either buy or sell the underlying financial instruments at a specified price on a specified future date. Although the buyer gets the rights to buy or sell the underlying options, there is no obligation to exercise this option. However, the seller of the contract is under an obligation to buy or sell the underlying instruments if the option is exercised.

Two types of financial options exist, namely call options and put options. Under a call option, the buyer of the contract gets the right to buy the financial instrument at the specified price at a future date, whereas a put option gives the buyer the right to sell the same at the specified price at the specified future date. The price that is paid by the buyer to the seller for exercising this level of flexibility is called the premium (the fair price). The prescribed future price is called the strike price.

The theoretical calculation of premium is connected namely with both the Black-Scholes Model (continuous statistical model based on normal distribution) and the Binomial or Trinomial Model (discrete statistical models based on binomial or trinomial distribution).

Financial options are either traded in an organized stock exchange or over-the-counter. The exchange traded options are known as standardized options. The options exchange is responsible for this standardization. This is done by specifying the quantity of the underlying financial instrument, its price and the future date of expiration. The details of these specifications may very vary from exchange to exchange. However, the broad outlines are similar.

Financial options are used either to hedge against risks by buying contracts that will pay out if something with negative financial consequences happens, or because it allows traders to magnify gains while limiting downside risks.

Financial options involve the risk of losing some or all of the contract price, if the market moves against the trend expected, and counterparty risk, such as broker insolvency or contractors who do not fulfill their contractual obligations.

2. Statistical Base of Financial Option

(quoted according to “mars.wiwi.hu-berlin.de/ebooks/html/sfe/sfenode41.html.” and “Zaskodny,P., Pavlat,V., Budik,J. (2007). Financial Derivates and Their Evaluation, Prague, Czech Republic: University of Finance and Administration”)

The Black-Scholes model traces the evolution of the option´s key underlying variables in continuous-time. This is done by means of both the standard normal probability densities $\rho(d_1)$, $\rho(d_2)$ and the standard normal distribution functions $N(d_1)$, $N(d_2)$.

The variables d_1, d_2 are connected with Spot price S, Strike price X, Risk-Free Rate r, Annual Dividend d, Time to Maturity τ, and Volatility σ.

The basic formulas for Black-Scholes model (Value Function – Fair Price for call option is marked “$\langle C \rangle$”, Value Function – Fair Price for put option is marked “$\langle P \rangle$”):

$$\langle C\rangle = Se^{-d\tau} N(d_1) - Xe^{-r\tau} N(d_2), \langle P\rangle = Xe^{-r\tau} N(-d_2) - Se^{-d\tau} N(-d_1)$$

$$d_1 = \frac{\ln \frac{S}{X} + \left(r - d + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}$$

$$N(d_1) = \int_{-\infty}^{d_1} \rho(d_1)\, d(d_1), N(d_2) = \int_{-\infty}^{d_2} \rho(d_2)\, d(d_2)$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}, \rho(d_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}}$$

The Binomial model traces the evolution of the option's key underlying variables in discrete-time. This is done by means of a binomial tree, for a number of time steps between the valuation and expiration dates (the number of time steps is marked “n”). Each node, in the tree, represents a possible price of the underlying at a given point in time.

At each step, it is assumed that the underlying instrument will move up or down by a specific factor (u or d) per step of the tree (where, by definition, $u \geq 1$ and $0 < d \leq 1$). So, if S is the spot price, then in the next period the price will either be $S_{up} = S.u$ or $S_{down} = S.d$.

The number of up factors is marked “j”, the number of down factors is “$n–j$”.

X is the Strike price and S is the Spot price of the underlying asset.

Under the risk neutrality assumption, today's fair price of a derivative is equal to the expected value of its future payoff discounted by the risk-free rate. Therefore, expected value is calculated using the option values from the later two nodes (Option up and Option down) weighted by their respective probabilities – "probability" p of an up move in the underlying, and "probability" **(**1-p**)** of a down move. The expected value is then discounted at q, the risk-free rate corresponding to the life of the option ($p = \frac{q-d}{u-d}$).

The basic formulas for Binomial model (Value Function – Fair Price for call option is marked “$\langle C \rangle$”, Value Function – Fair Price for put option is marked “$\langle P \rangle$”):

$$\langle C\rangle = \frac{1}{q^n} \sum_{j=0}^{n} \Pi_j C_j, \quad C_j = \max(0, S_j - X)$$

$$\langle P\rangle = \frac{1}{q^n}\sum_{j=0}^{n}\Pi_j P_j,\ P_j = \max(0, X - S_j)$$

$$\Pi_j = \binom{n}{j} p^j (1-p)^{n-j}$$

$$S_j = u^j d^{n-j} S,\ S_j^k = u^j d^{k-j} S$$

$$\binom{n}{k} = \frac{n!}{(n-k)!k!},\ m! = 1.2.......m$$

$$p = \frac{q-d}{u-d}, 1-p = \frac{u-q}{u-d}.$$

The Trinomial model traces the evolution of the option's key underlying variables in discrete-time. This is done by means of a trinomial tree, for a number of time steps between the valuation and expiration dates (the number of time steps is marked "*n*"). Each node, in the tree, represents a possible price of the underlying at a given point in time.

The fair price can be determined numerically. The Binomial model after Cox-Ross-Rubinstein can be used. In this section it will be introduced a less complex but numerically efficient approach based on trinomial trees. It is related to the classical numerical procedures for solving partial differential equations, which are also used to solve the Black-Scholes differential equations.

The Trinomial model follows the procedure of the binomial model whereby the price at each time step can change to three instead of two directions.

At each step, it is assumed that the underlying instrument will move up or down by a specific factor (e.g. two up factors u_1, u_2 and one down factor d) per step of the tree (where, by definition, $u_1, u_2 \geq 1$ and $0 < d \leq 1$). So, if S is the Spot price, then in the next period the price will either be $S_{u1} = S.u_1$, $S_{u2} = S.u_2$ or $S_d = S.d$. The probability with which the price moves from S to S_{u1}, S_{u2}, S_d is represented as p_1, p_2, p_3 ($p_1 + p_2 + p_3 = 1$).

The number of u_1 factors is marked "*j*", the number of u_2 factors is marked "*i*", and the number of *d* factors is "*n–j–i*".

The basic formulas for Trinomial model (Value Function – Fair Price for call option is marked "$\langle C\rangle$", Value Function – Fair Price for put option is marked "$\langle P\rangle$"):

$$\langle C\rangle = \frac{1}{q^n}\sum_{i=0}^{n}\sum_{j=0}^{n}\Pi_{ij} C_{ij},\ i + j = n_{\max}$$

$$C_{ij} = \max(0, S_{ij} - X)$$

$$S_{ij} = u_1^j u_2^i d^{n-i-j} S$$

$$\overline{S} = \sum_{i=0}^{n}\sum_{j=0}^{n}\Pi_{ij} S_{ij}, i + j = n_{\max}$$

$$\Pi_{ij} = \binom{n}{ij} p_1^i p_2^j (1 - p_1 - p_2)^{n-i-j}$$

$$\sum_{i=0}^{n}\sum_{j=0}^{n}\Pi_{ij} = 1, i + j = n_{\max}$$

$$\binom{n}{ij} = \frac{n!}{i!\,j!\,(n-i-j)!}$$

3. Greeks and Option Hedging in Black-Scholes and Binomial Models

(quoted according to http://en.wikipedia.org/wiki/Greeks_(finance))

In mathematical finance, the Greeks are the quantities representing the sensitivities of derivatives such as options to a change in underlying parameters on which the value function of an instrument or portfolio of financial instruments is dependent. The name is used because the most common of these sensitivities are often denoted by Greek letters. Collectively these have also been called the Risk Sensitivities, Risk Measures or Hedge Parameters.

The Greeks are vital tools in Risk Management. Each Greek measures the sensitivity of the value function of a financial instrument or portfolio to a small change in a given underlying parameter, so that component risks may be treated in isolation, and the portfolio rebalanced accordingly to achieve a desired exposure (see for example Delta Hedging).

The Greeks in the Black-Scholes model are relatively easy to calculate, a desirable property of financial models, and are very useful for derivatives traders, especially those who seek to hedge their portfolios from adverse changes in market conditions. For this reason, those Greeks which are particularly for Hedging Delta, Gamma and Vega are well-defined for measuring changes in Price, Time and Volatility.

The most common of the Greeks are the first order derivates: Delta, Dual Delta, Vega, Theta and Rho as well as Gamma, a second-order derivate of value function. Although Rho is a primary input into the Black-Scholes model, the overall impact on the value function of an option corresponding to changes in the risk-free rate is generally insignificant and therefore higher-order derivates involving the risk-free interest rate are not common.

The most used of the Greeks are some second order derivates: Gamma, Dual Gamma, Vomma, Vanna, Charm, DvegaDtime. Also the most used of the Greeks are some third order derivates: Speed, Zomma, Color, Ultima.

The Greeks in the Binomial model trace the evolution of the option's key underlying variables in discrete-time. The most used of the Greeks are the Delta and Gamma. Those Greeks are well-defined for Hedging Delta and Gamma.

The most common of the Greeks in the Black-Scholes and Binomial models are the Delta, Vega, Theta and Gamma. The most used of the Option Hedging are the Hedging Delta and Gamma. The remaining sensitivities (and hedging connected with them) in this list are common enough that they have common names, but this list is by no means exhaustive.

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4. Statistical Data Mining Tools – Normal, Binomial and Trinomial Distributions

4.1. Standard Normal Probability Density $\rho(x)$ and Standard Normal Distribution Function $N(x)$

$$N(x) = \int_{-\infty}^{x} \rho(x)\,dx$$

$$\rho(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

4.2. Binomial and Trinomial Probability Functions

$$\Pi_j = \binom{n}{j} p^j (1-p)^{n-j}$$

$$\Pi_{ij} = \binom{n}{ij} p_1^i p_2^j (1-p_1-p_2)^{n-i-j}$$

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5. Mathematical Data Mining Tools

5.1 Differential Calculus

Differentiation is a method to compute the rate at which a dependent output f changes with respect to the change in the independent input x. Given $f(x)$ a function of the one real variable x. If a change of x, Δx, causes the change of f, Δf, the average rate of change of f means $\Delta f / \Delta x$. The instantaneous rate of change of f, called derivate f', is obtained by

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \frac{\mathrm{d}f}{\mathrm{d}x}$$

where dx and df denote infinitesimal changes of x and f. In the case when f is a function of some another function $y(x)$, i.e. $f\left(y(x)\right)$, the derivate of f according to x has the form

$$f'(x) = \frac{\mathrm{d}f\left(y(x)\right)}{\mathrm{d}x} = \frac{\mathrm{d}f}{\mathrm{d}y}\frac{\mathrm{d}y}{\mathrm{d}x}.$$

The derivate $f'(x)$ can be again derived obtaining $f''(x)$ and so on.

If f is a function of several real variables, say $x_1,\ldots,x_n$, the rate of change of f corresponding to a change of variable x_1, held other variables fixed, is

$$\frac{\partial f}{\partial x_1} = \frac{\mathrm{d}f}{\mathrm{d}x_1}\left(x_2, ..., x_n = const.\right)$$

This result is the function of one variable and is called partial derivate of *f* with respect to x_1 and similarly for other variables. A complete picture by considering all variables at once is given by total differential

$$\mathrm{d}f\left(x_1, ..., x_n\right) = \frac{\partial f}{\partial x_1}\mathrm{d}x_1 + ... + \frac{\partial f}{\partial x_n}\mathrm{d}x_n .$$

5.2 Integral Calculus

The opposite procedure to the differentiation is the integration. The antiderivate (or indefinite integral or primitive function) to continuous function $f(x)$ is $F(x) + c$ (c being a constant) if and only if

$$\frac{\mathrm{d}F}{\mathrm{d}x} = f\left(x\right).$$

The difference

$$F\left(b\right) - F\left(a\right) = \int_a^b f\left(x\right)\mathrm{d}x$$

is the definite integral, where *a* and *b* are the lower and upper limit respectively.

The fundamental theorem of calculus sounds: Let *f* be a real-valued integrable function defined on a closed interval <*a*, *b*>. If *F* is defined for *x* in <*a*, *b*> by

$$F\left(x\right) = \frac{\mathrm{d}}{\mathrm{d}x}\int_a^x f\left(t\right)\mathrm{d}t ,$$

then *F* is continuous on <*a*, *b*>. It follows

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_a^x f\left(t\right)\mathrm{d}t = f\left(x\right).$$

Eventually, if the upper limit of the integral is a function $g(x)$,

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_a^{g(x)} f\left(t\right)\mathrm{d}t = f\left(g\left(x\right)\right)g'\left(x\right).$$

5.3 Convergent Progression and Convergent Series

The infinite progression of real numbers a_n, $n = 1,2,...$ is convergent if there exist finite real number a for which it holds

$$\lim_{n\to\infty} a_n = a .$$

An infinite series $\sum a_n$ is said to be convergent when the infinite progression s_n of partial sums has a finite limit, where

$$s_1 = a_1$$
$$s_1 = a_1 + a_2$$
$$s_1 = a_1 + a_2 + a_3$$
$$\dots$$

5.4 Taylor Series

The Taylor series is a power expansion of a function $f(x)$ about a point $x = a$

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

5.5 Binomial Series

The binomial theorem expressing the n-th power of a binom $(x + y)$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k ,$$

can be also expressed in the simplified form

$$(1-x)^n = \sum_{k=0}^{n} (-1)^k x^k ,$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

5.6 Trinomial Series

Similarly, the trinomial series is given by

$$(x_1 + x_2 + x_3)^n = \sum_{k_1,k_2,k_3} \binom{n}{k_1,k_2,k_3} x_1^{k_1} x_2^{k_2} x_3^{k_3} ,$$

Where

$$\binom{n}{k_1,k_2,k_3} = \frac{n!}{k_1!k_2!k_3!} .$$

It can be again brought simplified form as

$$\left(1-x_1-x_2\right)^n=\sum_{k_1,k_2}\binom{n}{k_1,k_2}\left(-1\right)^{k_1+k_2}\,x_1^{k_1}x_2^{k_2}.$$

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6. Conclusion

The Statistical Data Mining Tools in Greeks and Option Hedging (as a Part of Data Mining in Statistics Education):

- Normal distribution,
- Binomial distribution,
- Trinomial distribution.

The Mathematical Data Mining Tools in Greeks and Option Hedging (as a Part of Data Mining in Statistics Education):

- Infinitesimal calculus,
- Derivate of an integral according to its upper limit,
- Taylor series, Binomial series, Trinomial series.

Where will be used the partial data mining tools in statistics education? In Greeks

(***With the support of IGA VSFS 7727***)

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Abstract:

An imperative of data mining and a need of cooperation of the human with today´s computers are emphasized by D.A.Keim (Keim, 2002):

"The progress made in hardware technology allows today´s computer systems to store very large amounts of data. Researchers from the University of Berkeley estimate that every year 1 Exabyte (= 1 Million Terabyte) of data are generated, of which a large portion is available in digital form. This means that in the next three years more data will be generated than in all of human history before".

"If the data is presented textually, the amount of data which can be displayed is in range one hundred data items, but this is like a drop in the ocean when dealing with data sets containing millions of data items".

"For data mining to be effective, it is important to include the human in the data exploration process and combine the flexibility, creativity, and general knowledge of the human with the enormous storage capacity and the computational power of today´s computers."

The basic concepts about Data Mining in Statistics Education – see Záškodný,P., Tarábek,P. (2010-2011), ***Data Mining Tools in Statistics Education*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.). The complex data mining tool in statistics education is the Curricular Process of Statistics. The partial data mining tools in statistics education are the Analytical Synthetic Modelling, Statistical and Mathematical Data Mining Tools.

Description of Statistical and Mathematical Data Mining Tools in Statistics Education – see Záškodný,P., Havlíček,I., Budinský,P. (2010-2011), ***Partial Data Mining Tools in Statistics Education – in Greeks and Option Hedging*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk).

The statistical and mathematical data mining tools (as partial data mining tools) will be used in the Greeks and Option Hedging as a part of the statistics education. **The survey of the Greeks (based on the Black-Scholes Model) will be described in this paper.**

In mathematical finance, the Greeks are the quantities representing the sensitivities of derivates such as options to a change in underlying parameters on which the value function of an instrument or portfolio of financial instruments is dependent. The name is used because the most common of these sensitivities are often denoted by Greek letters.

The main principle of paper: Data Mining in Statistics Education (Greeks and Option Hedging)
The main goal of paper: Survey of Greeks
The procedure of paper: Value Function
Segmentation and Definitions of Greeks
Indications of Greeks
Formulas for Greeks
Needful Relations for Deduction of Greeks Formulas

The results of paper:

1. *Value Function as Fair Price (as Premium)*
2. *Greeks of First Order*
3. *Greeks of Second Order*
4. *Greeks of Third Order*
5. *Names of Greeks*
6. *Survey of Formulas for Greeks Calculation*
7. *Survey of Needful Relations*

Key Words:

Data Mining, Statistics Education, Partial Data Mining Tool – Statistical Data Mining Tools and Mathematical Data Mining Tools, Financial Options, Greeks, Option Hedging, Black-Scholes Model, Value Function, Greeks of First, Second and Third Order, Names of Greeks, Formulas for Greeks, Needful Relations for Deduction of Greeks Formulas

1. Value Function

(quoted according to Záškodný,P., Havlíček,I., Budinský,P. (2010-2011), ***Partial Data Mining Tools in Statistics Education – in Greeks and option Hedging*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.)

Financial options are those derivative contracts in which the underlying assets are financial instruments such as stocks, bonds or an interest rate. The options on financial instruments provide a buyer with the right to either buy or sell the underlying financial instruments at a specified price on a specified future date. Although the buyer gets the rights to buy or sell the underlying options, there is no obligation to exercise this option. However, the seller of the contract is under an obligation to buy or sell the underlying instruments if the option is exercised.

Two types of financial options exist, namely call options and put options. Under a call option, the buyer of the contract gets the right to buy the financial instrument at the specified price at a future date, whereas a put option gives the buyer the right to sell the same at the specified price at the specified future date. The price that is paid by the buyer to the seller for exercising this level of flexibility is called the premium (the fair price, **the value function**). The prescribed future price is called the strike price.

The theoretical calculation of premium is connected namely with both the Black-Scholes Model (continuous statistical model based on normal distribution) and the Binomial or Trinomial Model (discrete statistical models based on binomial or trinomial distribution). **In this paper the priority will be given to Black-Scholes Model.**

The Black-Scholes model traces the evolution of the option´s key underlying variables in continuous-time. This is done by means of both the standard normal probability densities $\rho(d_1)$, $\rho(d_2)$ and the standard normal distribution functions $N(d_1)$, $N(d_2)$.

The variables d_1, d_2 are connected with Spot price S, Strike price X, Risk-Free Rate r, Annual Dividend d, Time to Maturity τ, Volatility σ, and Annual Dividend Yield d.

Value Function *V* (as Fair Price or as Premium) can be expressed as a function of five quantities $V = \mathrm{f}\,(S, X, r, \tau, \sigma)$

The basic formulas for Black-Scholes model (Value Function V – Fair Price for call option is marked “$\langle C \rangle$”, Value Function – Fair Price for put option is marked “$\langle P \rangle$”):

$$\langle C\rangle = Se^{-d\tau} N(d_1) - Xe^{-r\tau} N(d_2), \langle P\rangle = Xe^{-r\tau} N(-d_2) - Se^{-d\tau} N(-d_1)$$

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}$$

$$N(d_1) = \int_{-\infty}^{d_1} \rho(d_1)\, d(d_1), N(d_2) = \int_{-\infty}^{d_2} \rho(d_2)\, d(d_2)$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}, \rho(d_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}}$$

2. Segmentation and Definitions of Greeks

2.1. Greeks of First Order

The speeds of value function change:

$$\Delta = \frac{\partial V}{\partial S}$$

$$\text{Dual}\,\Delta = \frac{\partial V}{\partial X}$$

$$\nu(\text{vega}) = \frac{\partial V}{\partial \sigma}$$

$$\Theta = -\frac{\partial V}{\partial \tau}$$

$$\rho = \frac{\partial V}{\partial r}$$

2.2. Greeks of Individual Second Order

The accelerations of value function change & the speeds of first order greeks change:

$$\Gamma = \frac{\partial^2 V}{\partial S^2}$$

$$\text{Dual}\,\Gamma = \frac{\partial^2 V}{\partial X^2}$$

$$\text{Vomma} = \frac{\partial^2 V}{\partial \sigma^2}$$

$$\text{Out of Use} = \frac{\partial^2 V}{\partial \tau^2}$$

$$\text{Out of Use} = \frac{\partial^2 V}{\partial r^2}$$

2.3. Greeks of Combined Second Order

The speeds of first order greeks change:

$$\text{Vanna} = \frac{\partial^2 V}{\partial S\,\partial \sigma}$$

$$\text{Charm} = \frac{\partial^2 V}{\partial S\,\partial \tau}$$

$$\text{DvegaDtime} = \frac{\partial^2 V}{\partial \sigma\,\partial \tau}$$

2.4. Greeks of Third Order

The speeds of second order greeks change:

$$\text{Speed} = \frac{\partial^3 V}{\partial S^3}$$

$$\text{Zomma} = \frac{\partial^3 V}{\partial S^2 \partial \sigma}$$

$$\text{Color} = \frac{\partial^3 V}{\partial S^2 \partial \tau}$$

$$\text{Ultima} = \frac{\partial^3 V}{\partial \sigma^3}$$

3. Indications of Greeks

3.1. Greeks of First Order

$$\Delta = \frac{\partial V}{\partial S} = \text{DvalueDspot}$$

$$\text{Dual}\,\Delta = \frac{\partial V}{\partial X} = \text{DvalueDstrike}$$

$$\nu\left(\text{Vega}\right) = \frac{\partial V}{\partial \sigma} = \text{DvalueDvol}$$

$$\Theta = -\frac{\partial V}{\partial \tau} = -\text{DvalueDtime}$$

$$\rho = \frac{\partial V}{\partial r} = \text{DvalueDrate}$$

3.2. Greeks of Second Order

$$\Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{\partial \Delta}{\partial S} = \text{DdeltaDspot}$$

$$\text{Dual}\,\Gamma = \frac{\partial^2 V}{\partial X^2} = \frac{\partial \text{Dual}\Delta}{\partial \text{X}} = \text{DdualdeltaDstrike}$$

$$\text{Vomma} = \frac{\partial^2 V}{\partial \sigma^2} = \frac{\partial \nu}{\partial \sigma} = \text{DvegaDvol}$$

$$\text{Vanna} = \frac{\partial^2 V}{\partial S\,\partial \sigma} = \frac{\partial \Delta}{\partial \sigma} = \frac{\partial \nu}{\partial S} = \text{DdeltaDvol} = \text{DvegaDspot}$$

$$\text{Charm} = \frac{\partial^2 V}{\partial S\,\partial \tau} = \frac{\partial \Delta}{\partial \tau} = \frac{\partial(-\Theta)}{\partial \text{S}} = \text{DdeltaDtime} = \text{D}(-\text{theta})\text{Dspot}$$

$$\text{DvegaDtime} = \frac{\partial^2 V}{\partial \sigma \partial \tau} = \frac{\partial(-\Theta)}{\partial \sigma} = \frac{\partial \nu}{\partial \tau} = \text{D}(-\text{theta})\text{Dvol} = \text{DvegaDtime}$$

3.3. Greeks of Third Order

$$\text{Speed} = \frac{\partial^3 V}{\partial S^3} = \frac{\partial \Gamma}{\partial S} = \frac{\partial^2 \Delta}{\partial S^2} = \text{DgammaDspot}$$

$$\text{Zomma} = \frac{\partial^3 V}{\partial S^2 \partial \sigma} = \frac{\partial \Gamma}{\partial \sigma} = \frac{\partial^2 \Delta}{\partial S\,\partial \sigma} = \frac{\partial^2 \nu}{\partial S^2} = \text{DgammaDvol}$$

$$\text{Color} = \frac{\partial^3 V}{\partial S^2 \partial \tau} = \frac{\partial \Gamma}{\partial \tau} = \frac{\partial^2 \Delta}{\partial S\,\partial \tau} = \frac{\partial^2(-\Theta)}{\partial S^2} = \text{DgammaDtime}$$

$$\text{Ultima} = \frac{\partial^3 V}{\partial \sigma^3} = \frac{\partial \text{vomma}}{\partial \sigma} = \frac{\partial^2 \nu}{\partial \sigma^2} = \text{DvommaDvol}$$

4. Formulas for Greeks (CO – Call Option, PO – Put Option)

4.1. Formulas for Delta Greek Δ

$$\Delta_{CO} = e^{-d\tau} N(d_1)$$
$$\Delta_{PO} = -e^{-d\tau} N(-d_1)$$

4.2. Formulas for Dual Delta Greek Dual Δ

$$\text{Dual}\,\Delta_{CO} = -e^{-r\tau} N(d_2)$$
$$\text{Dual}\,\Delta_{PO} = e^{-r\tau} N(-d_2)$$

4.3. Formulas for Vega Greek ν

$$\nu_{CO,PO} = e^{-d\tau} S\rho(d_1)\sqrt{\tau} = Xe^{-r\tau}\rho(d_2)\sqrt{\tau}$$

4.4. Formulas for Theta Greek Θ

$$\Theta_{CO} = -e^{-d\tau}\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}} - rXe^{-r\tau}N(d_2)$$

$$\Theta_{PO} = -e^{-d\tau}\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}} + rXe^{-r\tau}N(-d_2)$$

4.5. Formulas for Rho Greek ρ

$$\rho_{CO} = \tau Xe^{-r\tau}N(d_2)$$

$$\rho_{PO} = -\tau Xe^{-r\tau}N(-d_2)$$

4.6. Formula for Gamma Greek Γ

$$\Gamma_{CO,PO} = e^{-d\tau}\frac{\rho(d_1)}{S\sigma\sqrt{\tau}}$$

4.7. Formula for Dual Gamma Greek Dual Γ

$$\text{Dual}\,\Gamma_{CO,PO} = e^{-r\tau}\frac{\rho(d_2)}{X\sigma\sqrt{\tau}}$$

4.8. Formulas for Vomma Greek Vomma

$$\text{Vomma}_{CO,PO} = Se^{-d\tau}\rho(d_1)\sqrt{\tau}\frac{d_1 d_2}{\sigma} = \nu\frac{d_1 d_2}{\sigma}$$

4.9. Formulas for Vanna Greek Vanna

$$\text{Vanna}_{CO,PO} = -e^{-d\tau}\rho(d_1)\frac{d_2}{\sigma} = -\frac{\nu}{S}\frac{d_2}{\sigma\sqrt{\tau}} = \frac{\nu}{S}\left(1 - \frac{d_1}{\sigma\sqrt{\tau}}\right)$$

4.10. Formulas for Charm Greek Charm

$$\text{Charm}_{CO} = -de^{-d\tau}N\left(d_1\right)+e^{-d\tau}\rho\left(d_1\right)\frac{2\left(r-d\right)\tau-d_2\sigma\sqrt{\tau}}{2\sigma\tau\sqrt{\tau}}$$

$$\text{Charm}_{PO} = de^{-d\tau}N\left(-d_1\right)+e^{-d\tau}\rho\left(d_1\right)\frac{2\left(r-d\right)\tau-d_2\sigma\sqrt{\tau}}{2\sigma\tau\sqrt{\tau}}$$

4.11. Formulas for DvegaDtime Greek DvegaDtime

$$\text{DvegaDtime}_{CO,PO} = -e^{-d\tau}S\rho\left(d_1\right)\sqrt{\tau}\left(d+\frac{\left(r-d\right)d_1}{\sigma\sqrt{\tau}}-\frac{1+d_1d_2}{2\tau}\right)$$

$$\text{DvegaDtime}_{CO,PO} = -\nu\left(d+\frac{\left(r-d\right)d_1}{\sigma\sqrt{\tau}}-\frac{1+d_1d_2}{2\tau}\right)$$

4.12. Formulas for Speed Greek Speed

$$\text{Speed}_{CO,PO} = -e^{-d\tau}\frac{\rho\left(d_1\right)}{S^2\sigma\sqrt{\tau}}\left(\frac{d_1}{\sigma\sqrt{\tau}}+1\right)=-\frac{\Gamma}{S}\left(\frac{d_1}{\sigma\sqrt{\tau}}+1\right)$$

4.13. Formulas for Zomma Greek Zomma

$$\text{Zomma}_{CO,PO} = e^{-d\tau}\frac{\rho\left(d_1\right)}{S\sigma^2\sqrt{\tau}}\left(d_1d_2-1\right)=\Gamma\left(\frac{d_1d_2-1}{\sigma}\right)$$

4.14. Formulas for Color Greek Color

$$\text{Color}_{CO,PO} = -e^{-d\tau}\frac{\rho\left(d_1\right)}{2S\sigma\tau\sqrt{\tau}}\left(2d\tau+1+\frac{2\left(r-d\right)\tau-d_2\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}d_1\right)$$

$$\text{Color}_{CO,PO} = -\frac{\Gamma}{2\tau}\left(2d\tau+1+\frac{2\left(r-d\right)\tau-d_2\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}d_1\right)$$

4.15. Formulas for Ultima Greek Ultima

$$\text{Ultima}_{CO,PO} = e^{-d\tau}\frac{S\rho\left(d_1\right)\sqrt{\tau}}{\sigma^2}\left(d_1d_2\left(d_1d_2\sqrt{\tau}-2\sigma-1\right)+\sigma\sqrt{\tau}\left(d_2-d_1\right)\right)$$

$$\text{Ultima}_{CO,PO} = \frac{\nu}{\sigma^2}\left(d_1d_2\left(d_1d_2\sqrt{\tau}-2\sigma-1\right)+\sigma\sqrt{\tau}\left(d_2-d_1\right)\right)$$

5. Needful Relations for Deduction of Greeks Formulas

5.1. Value Function

$$\langle C\rangle = Se^{-d\tau} N\left(d_1\right) - Xe^{-r\tau} N\left(d_2\right), \langle P\rangle = Xe^{-r\tau} N\left(-d_2\right) - Se^{-d\tau} N\left(-d_1\right)$$

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}, d_2 = \frac{\ln S/X + \left(r - d - \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

5.2. Standard Normal Probability Densities

$$\rho\left(d_1\right) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}, \rho\left(d_2\right) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}}$$

$$\rho\left(d_1\right) = \rho\left(d_2\right) e^{-d_2\sigma\sqrt{\tau}} e^{-\tau\sigma^2/2}, \rho\left(d_2\right) = \rho\left(d_1\right) e^{d_1\sigma\sqrt{\tau}} e^{-\tau\sigma^2/2}$$

$$e^{d_1\sigma\sqrt{\tau}} = \frac{S}{X} e^{\left(r+\sigma^2/2\right)\tau} e^{-d\tau}, e^{-d_2\sigma\sqrt{\tau}} = \frac{S}{X} e^{-\left(r-\sigma^2/2\right)\tau} e^{d\tau}$$

5.3. Standard Normal Distribution Functions

$$N\left(d_1\right) = \int_{-\infty}^{d_1} \rho\left(d_1\right) d\left(d_1\right), N\left(d_2\right) = \int_{-\infty}^{d_2} \rho\left(d_2\right) d\left(d_2\right)$$

$$N\left(d_1\right) + N\left(-d_1\right) = 1, N\left(d_2\right) + N\left(-d_2\right) = 1$$

$$\frac{\partial N\left(d_1\right)}{\partial d_1} = \rho\left(d_1\right), \frac{\partial N\left(d_2\right)}{\partial d_2} = \rho\left(d_2\right)$$

6. Conclusion

The results of paper:

- Description of Value Function as Fair Price
- Description of Greeks of First Order
- Description of Greeks of Second Order
- Description of Greeks of Third Order
- Names and Indications of Greeks
- Survey of Formulas for Greeks Calculation
- Survey of Needful Relations for Greeks Calculation

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Application of Mathematical Data Mining Tools – Greeks of First Order (Delta, Dual Delta)

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Abstract:

An imperative of data mining and a need of cooperation of the human with today´s computers are emphasized by D.A.Keim (Keim, 2002):

"The progress made in hardware technology allows today´s computer systems to store very large amounts of data. Researchers from the University of Berkeley estimate that every year 1 Exabyte (= 1 Million Terabyte) of data are generated, of which a large portion is available in digital form. This means that in the next three years more data will be generated than in all of human history before".

"If the data is presented textually, the amount of data which can be displayed is in range one hundred data items, but this is like a drop in the ocean when dealing with data sets containing millions of data items".

"For data mining to be effective, it is important to include the human in the data exploration process and combine the flexibility, creativity, and general knowledge of the human with the enormous storage capacity and the computational power of today´s computers."

The basic concepts about Data Mining in Statistics Education – see Záškodný,P., Tarábek.P. (2010-2011), ***Data Mining Tools in Statistics Education*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.) The complex data mining tool in statistics education is the Curricular Process of Statistics. The partial data mining tools in statistics education are the Analytical Synthetic Modelling, Statistical and Mathematical Data Mining Tools.

Description of Statistical and Mathematical Data Mining Tools in Statistics Education – see Záškodný,P., Havlíček,I., Budinský,P. (2010-2011), ***Partial Data Mining Tools in Statistics Education – in Greeks and Option Hedging*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.)

Description of Greeks of First Order in Statistics Education – see Záškodný,P., Havlíček,I., Budinský,P., Hrdlička,L. (2010-2011), ***Where will be used the partial data mining tools in statistics education? In Greeks*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.)

The application of mathematical data mining tools in the light of a deduction of the first order greeks Delta and Dual Delta (based on the Black-Scholes Model) will be described in this paper.

The main principle of paper: Mathematical Data Mining Tools in Statistics Education
The main goal of paper: Deduction of Greeks of First Order
The procedure of paper: Survey of Greeks of First order
Deduction of Delta Greek
Deduction of Dual Delta Greek
Exercising Delta Greek
Exercising Dual Delta Greek

The results of paper:

1. *Basic Mathematical Derivation of Delta Greek*
2. *Mathematical Derivation of Lambda Greek*
3. *Basic Mathematical Derivation of Dual Delta Greek*
4. *Partial Mathematical Derivation of Delta Greek*
5. *Partial Mathematical Derivation of Dual Delta Greek*

Key Words:

Data Mining, Statistics Education, Partial Data Mining Tool – Mathematical Data Mining Tools, Black-Scholes Model, Value Function, Greeks of First Order, Delta Greek, Dual Delta Greek, Lambda Greek, Derivation of Formulas for First Order Greeks "Delta" and "Dual Delta"

1. Survey of Greeks of First Order

The Black-Scholes model traces the evolution of the option´s key underlying variables in continuous-time. This is done by means of both the standard normal probability densities $\rho(d_1)$, $\rho(d_2)$ and the standard normal distribution functions $N(d_1)$, $N(d_2)$.

The variables d_1, d_2 are connected with Spot price S, Strike price X, Risk-Free Rate r, Annual Dividend Yield d, Time to Maturity τ, and Volatility σ.

Value Function *V* (as Fair Price or as Premium) can be expressed as a function of five quantities $V = \mathrm{f}\,(S, X, r, \tau, \sigma)$

Survey of greeks of the first order as survey of the speeds of value function change (see Záškodný,P., Havlíček,I., Budinský,P., Hrdlička,L. (2010-2011):

$$\Delta = \frac{\partial V}{\partial S} = \text{DvalueDspot} = \text{Delta Greek}$$

$$\text{Dual}\,\Delta = \frac{\partial V}{\partial X} = \text{DvalueDstrike} = \text{Dual Delta Greek}$$

$$\Theta = -\frac{\partial V}{\partial \tau} = -\text{DvalueDtime} = \text{Theta Greek}$$

$$\rho = \frac{\partial V}{\partial r} = \text{DvalueDrate} = \text{Rho Greek}$$

$$\nu\left(\text{Vega}\right) = \frac{\partial V}{\partial \sigma} = \text{DvalueDvol} = \text{Vega Greek}$$

2. Mathematical Derivation of Delta Greek (DvalueDspot)

2.1. Basic Relations

$$\Delta = \frac{\partial V}{\partial S}, V = \langle C \rangle, V = \langle P \rangle$$

$$\langle C \rangle = Se^{-d\tau} N(d_1) - Xe^{-r\tau} N(d_2), \langle P \rangle = Xe^{-r\tau} N(-d_2) - Se^{-d\tau} N(-d_1)$$

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}$$

$$N(d_1) = \int_{-\infty}^{d_1} \rho(d_1) d(d_1), N(d_2) = \int_{-\infty}^{d_2} \rho(d_2) d(d_2)$$

$$N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}, \rho(d_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}}, \rho(d_2) = \rho(d_1) e^{d_1\sigma\sqrt{\tau}} e^{-\tau\sigma^2/2}$$

2.2. Implementation of Basic Derivate (without Annual Dividend Yield *d*)

$$\Delta = \frac{\partial C}{\partial S} = N(d_1) + S\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial S} - Xe^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial S}$$

2.3. Implementation of Partial Derivates

$$\frac{\partial N(d_1)}{\partial d_1} = \rho(d_1), \frac{\partial N(d_2)}{\partial d_2} = \rho(d_2)$$

$$\frac{\partial d_1}{\partial S} = \frac{\partial d_2}{\partial S} = \frac{\frac{X}{S}\frac{1}{X}}{\sigma\sqrt{\tau}} = \frac{1}{S\sigma\sqrt{\tau}}$$

2.4. Adjustment of Implemented Basic Derivate (without Annual Dividend Yield *d*)

$$S\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial S} = S\rho(d_1)\frac{1}{S\sigma\sqrt{\tau}}, Xe^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial S} = Xe^{-r\tau}\rho(d_2)\frac{1}{S\sigma\sqrt{\tau}}$$

$$S\rho(d_1) = Xe^{-r\tau}\rho(d_2) \Rightarrow \frac{S}{X} = e^{-r\tau}e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2} \Rightarrow$$

$$\Rightarrow \frac{S}{X} = e^{-r\tau}e^{-\frac{\tau\sigma^2}{2}} \exp\left(\frac{\ln S/X \,\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}\right)\exp\left(\frac{\left(r + \sigma^2/2\right)\tau\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}\right) \Rightarrow 1 = 1$$

2.5. Summary

Formulas for Delta Greek (Call Option without Annual Dividend Yield *d* and with Annual Dividend Yield *d*):

$$\Delta_{CO} = N\left(d_1\right), \Delta_{CO} = e^{-d\tau} N\left(d_1\right)$$

Formulas for Delta Greek (Put Option without Annual Dividend Yield *d* and with Annual Dividend Yield *d*, and Relations $N\left(d_1\right)+N\left(-d_1\right)=1, N\left(d_2\right)+N\left(-d_2\right)=1$):

$$\Delta_{PO} = -N\left(-d_1\right), \Delta_{PO} = -e^{-d\tau} N\left(-d_1\right)$$

2.6. Exercising on Derivation of Delta Greek

2.6.1. Deduce Relation for Delta Greek with Annual Dividend Yield *d* (Call Option)

Implementation of a basic derivate (see paragraph 2.2.with an annual dividend yield *d*)

$$\Delta = \frac{\partial C}{\partial S} = e^{-d\tau} N\left(d_1\right) + S e^{-d\tau} \frac{\partial N\left(d_1\right)}{\partial d_1} \frac{\partial d_1}{\partial S} - X e^{-r\tau} \frac{\partial N\left(d_2\right)}{\partial d_2} \frac{\partial d_2}{\partial S}$$

Adjustment of the basic derivate (see paragraph 2.4. with an annual dividend yield *d*)

$$S e^{-d\tau} \frac{\partial N\left(d_1\right)}{\partial d_1} \frac{\partial d_1}{\partial S} = S e^{-d\tau} \rho\left(d_1\right) \frac{1}{S\sigma\sqrt{\tau}},\ X e^{-r\tau} \frac{\partial N\left(d_2\right)}{\partial d_2} \frac{\partial d_2}{\partial S} = X e^{-r\tau} \rho\left(d_2\right) \frac{1}{S\sigma\sqrt{\tau}}$$

$$S e^{-d\tau} \rho\left(d_1\right) = X e^{-r\tau} \rho\left(d_2\right) \Rightarrow \frac{S}{X} = e^{-r\tau} e^{d_1\sigma\sqrt{\tau}} e^{-\tau\sigma^2} e^{d\tau} \Rightarrow$$

$$\Rightarrow \frac{S}{X} = e^{-r\tau} e^{-\frac{\tau\sigma^2}{2}} \exp\left(\frac{\ln S/X\ \sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}\right) \exp\left(\frac{\left(r+\sigma^2/2\right)\tau\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}\right) e^{-\frac{d\tau\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}} e^{d\tau} \Rightarrow 1=1$$

Result:

$$\Delta = e^{-d\tau} N\left(d_1\right)$$

2.6.2. Deduce Relation for Delta Greek without Annual Dividend Yield *d* (Put Option)

Implementation of the basic derivate (see paragraph 2.2. with adjustment on a put option)

$$\Delta = \frac{\partial P}{\partial S} = -N\left(-d_1\right) - S \frac{\partial N\left(-d_1\right)}{\partial d_1} \frac{\partial d_1}{\partial S} + X e^{-r\tau} \frac{\partial N\left(-d_2\right)}{\partial d_2} \frac{\partial d_2}{\partial S}$$

Application of the relations $N\left(d_1\right)+N\left(-d_1\right)=1, N\left(d_2\right)+N\left(-d_2\right)=1$ (see paragraph 2.1.), application of the adjustments implemented in paragraph 2.4. – final adjustment of implemented basic derivative:

$$\Delta = \frac{\partial P}{\partial S} = -N(-d_1) - S\frac{\partial(1-N(d_1))}{\partial d_1}\frac{\partial d_1}{\partial S} + Xe^{-r\tau}\frac{\partial(1-N(d_2))}{\partial d_2}\frac{\partial d_2}{\partial S}$$

$$\Delta = \frac{\partial P}{\partial S} = -N(-d_1) + S\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial S} - Xe^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial S}$$

$$S\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial S} = Xe^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial S} \Rightarrow 1 = 1$$

Result:

$$\Delta = -N(-d_1)$$

2.6.3. Deduce Relation for Delta Greek with Annual Dividend Yield *d* (Put Option)

With capitalization of paragraphs 2.6.1., 2.6.2., it is possible gradually to write

$$\Delta = \frac{\partial P}{\partial S} = -e^{-d\tau}N(-d_1) - Se^{-d\tau}\frac{\partial N(-d_1)}{\partial d_1}\frac{\partial d_1}{\partial S} + Xe^{-r\tau}\frac{\partial N(-d_2)}{\partial d_2}\frac{\partial d_2}{\partial S}$$

$$\Delta = \frac{\partial P}{\partial S} = -e^{-d\tau}N(-d_1) - Se^{-d\tau}\frac{\partial(1-N(d_1))}{\partial d_1}\frac{\partial d_1}{\partial S} + Xe^{-r\tau}\frac{\partial(1-N(d_2))}{\partial d_2}\frac{\partial d_2}{\partial S}$$

$$\Delta = \frac{\partial P}{\partial S} = -e^{-d\tau}N(-d_1) + Se^{-d\tau}\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial S} - Xe^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial S}$$

$$Se^{-d\tau}\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial S} = Se^{-d\tau}\rho(d_1)\frac{1}{S\sigma\sqrt{\tau}},\ Xe^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial S} = Xe^{-r\tau}\rho(d_2)\frac{1}{S\sigma\sqrt{\tau}}$$

$$Se^{-d\tau}\rho(d_1) = Xe^{-r\tau}\rho(d_2) \Rightarrow \frac{S}{X} = e^{-r\tau}e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2}e^{d\tau} \Rightarrow$$

$$\Rightarrow \frac{S}{X} = e^{-r\tau}e^{-\frac{\tau\sigma^2}{2}}\exp\left(\frac{\ln S/X\ \sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}\right)\exp\left(\frac{\left(r+\sigma^2/2\right)\tau\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}\right)e^{-\frac{d\tau\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}}e^{d\tau} \Rightarrow 1 = 1$$

Result:

$$\Delta = -e^{-d\tau}N(-d_1)$$

2.6.4. Present Relation for Lambda Greek (Elasticity) without/with an annual dividend yield (call option, put option)

Lambda Greek λ is defined as Delta Greek Δ multiplied by portion $\frac{S}{V}$, where $V = \langle C\rangle, V = \langle P\rangle$. According to paragraph 2.5. the formulas for Lambda Greek:

a) Call option without/with an annual dividend yield *d*: $\lambda_{CO} = N(d_1)\frac{S}{C}, \lambda_{CO} = e^{-d\tau}N(d_1)\frac{S}{C}$

b) Put option without/with an annual dividend yield *d*: $\lambda_{PO} = -N(-d_1)\frac{S}{P}, \lambda_{PO} = -e^{-d\tau}N(-d_1)\frac{S}{P}$

(see relations $N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$)

3. Mathematical Derivation of Dual Delta Greek (DvalueDstrike)

3.1. Basic Relations

$$Dual\ \Delta = \frac{\partial V}{\partial X}, V = \langle C \rangle, V = \langle P \rangle$$
$$\langle C \rangle = Se^{-d\tau} N(d_1) - Xe^{-r\tau} N(d_2), \langle P \rangle = Xe^{-r\tau} N(-d_2) - Se^{-d\tau} N(-d_1)$$
$$d_1 = \frac{\ln {S}/{X} + \left(r - d + {\sigma^2}/{2}\right)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}$$
$$N(d_1) = \int_{-\infty}^{d_1} \rho(d_1)\, d(d_1), N(d_2) = \int_{-\infty}^{d_2} \rho(d_2)\, d(d_2)$$
$$N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$$
$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}, \rho(d_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}}, \rho(d_2) = \rho(d_1) e^{d_1\sigma\sqrt{\tau}} e^{-\tau\sigma^2/2}$$

3.2. Implementation of Basic Derivate (without Annual Dividend Yield *d*)

$$Dual\ \Delta = \frac{\partial C}{\partial X} = S\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial X} - Xe^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial X} - e^{-r\tau} N(d_2)$$

3.3. Implementation of Partial Derivates

$$\frac{\partial N(d_1)}{\partial d_1} = \rho(d_1), \frac{\partial N(d_2)}{\partial d_2} = \rho(d_2)$$
$$\frac{\partial d_1}{\partial X} = \frac{\partial d_2}{\partial X} = \frac{\frac{X}{S} S \frac{-1}{X^2}}{\sigma\sqrt{\tau}} = \frac{-1}{X\sigma\sqrt{\tau}}$$

3.4. Adjustment of Implemented Basic Derivate (without Annual Dividend Yield *d*)

$$S\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial X} = S\rho(d_1)\frac{-1}{X\sigma\sqrt{\tau}}, Xe^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial X} = Xe^{-r\tau}\rho(d_2)\frac{-1}{X\sigma\sqrt{\tau}}$$
$$S\rho(d_1) = Xe^{-r\tau}\rho(d_2) \Rightarrow \frac{S}{X} = e^{-r\tau} e^{d_1\sigma\sqrt{\tau}} e^{-\tau\sigma^2} \Rightarrow$$
$$\Rightarrow \frac{S}{X} = e^{-r\tau} e^{-\frac{\tau\sigma^2}{2}} \exp\left(\frac{\ln {S}/{X}\, \sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}\right) \exp\left(\frac{\left(r + {\sigma^2}/{2}\right)\tau\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}\right) \Rightarrow 1 = 1$$

3.5. Summary

Formulas for Dual Delta Greek (Call Option without Annual Dividend Yield *d* and with Annual Dividend Yield *d*):

$$Dual\ \Delta_{CO} = \frac{\partial C}{\partial X} = -e^{-r\tau} N\left(d_2\right)$$

Formulas for Dual Delta Greek (Put Option without Annual Dividend Yield *d* and with Annual Dividend Yield *d*, and Relations $N\left(d_1\right) + N\left(-d_1\right) = 1,\ N\left(d_2\right) + N\left(-d_2\right) = 1$):

$$Dual\ \Delta_{PO} = \frac{\partial P}{\partial X} = e^{-r\tau} N\left(-d_2\right)$$

3.6. Exercising on Derivation of Dual Delta Greek

3.6.1. Deduce Relation for Dual Delta Greek with Annual Dividend Yield *d* (Call Option)

Implementation of a basic derivate (with an annual dividend yield *d*)

$$Dual\ \Delta = \frac{\partial C}{\partial X} = e^{-d\tau} S \frac{\partial N\left(d_1\right)}{\partial d_1} \frac{\partial d_1}{\partial X} - X e^{-r\tau} \frac{\partial N\left(d_2\right)}{\partial d_2} \frac{\partial d_2}{\partial X} - e^{-r\tau} N\left(d_2\right)$$

Implementation of the partial derivates

$$\frac{\partial N\left(d_1\right)}{\partial d_1} = \rho\left(d_1\right), \frac{\partial N\left(d_2\right)}{\partial d_2} = \rho\left(d_2\right)$$

$$\frac{\partial d_1}{\partial X} = \frac{\partial d_2}{\partial X} = \frac{\frac{X}{S} S \frac{-1}{X^2}}{\sigma\sqrt{\tau}} = \frac{-1}{X\sigma\sqrt{\tau}}$$

Adjustment of the basic derivate (with an annual dividend yield *d*)

$$e^{-d\tau} S \frac{\partial N\left(d_1\right)}{\partial d_1} \frac{\partial d_1}{\partial X} = e^{-d\tau} S \rho\left(d_1\right) \frac{-1}{X\sigma\sqrt{\tau}},$$

$$X e^{-r\tau} \frac{\partial N\left(d_2\right)}{\partial d_2} \frac{\partial d_2}{\partial X} = X e^{-r\tau} \rho\left(d_2\right) \frac{-1}{X\sigma\sqrt{\tau}}$$

$$e^{-d\tau} S \rho\left(d_1\right) = X e^{-r\tau} \rho\left(d_2\right) \Rightarrow e^{-d\tau} \frac{S}{X} = e^{-r\tau} e^{d_1\sigma\sqrt{\tau}} e^{-\tau\sigma^2} \Rightarrow$$

$$\Rightarrow e^{-d\tau} \frac{S}{X} = e^{-r\tau} e^{-\frac{\tau\sigma^2}{2}} \exp\left(\frac{\ln{}^{S}\!/_{X}\, \sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}\right) \exp\left(\frac{\left(r - d + {}^{\sigma^2}\!/_{2}\right)\tau\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}\right) \Rightarrow 1 = 1$$

Result:

$$Dual\ \Delta_{CO} = \frac{\partial C}{\partial X} = -e^{-r\tau} N\left(d_2\right)$$

3.6.2. Deduce Relation for Dual Delta Greek without Annual Dividend Yield *d* (Put Option)

Implementation of the basic derivate (with adjustment on a put option)

$$Dual\ \Delta = \frac{\partial P}{\partial X} = -S\frac{\partial N(-d_1)}{\partial d_1}\frac{\partial d_1}{\partial X} + Xe^{-r\tau}\frac{\partial N(-d_2)}{\partial d_2}\frac{\partial d_2}{\partial X} + e^{-r\tau}N(-d_2)$$

Implementation of the partial derivates

$$\frac{\partial N(d_1)}{\partial d_1} = \rho(d_1), \frac{\partial N(d_2)}{\partial d_2} = \rho(d_2)$$

$$\frac{\partial d_1}{\partial X} = \frac{\partial d_2}{\partial X} = \frac{\frac{X}{S} S \frac{-1}{X^2}}{\sigma\sqrt{\tau}} = \frac{-1}{X\sigma\sqrt{\tau}}$$

Adjustment of the basic derivate (without an annual dividend yield *d*)

$$S\frac{\partial N(-d_1)}{\partial d_1}\frac{\partial d_1}{\partial X} = -S\rho(d_1)\frac{-1}{X\sigma\sqrt{\tau}},$$

$$Xe^{-r\tau}\frac{\partial N(-d_2)}{\partial d_2}\frac{\partial d_2}{\partial X} = -Xe^{-r\tau}\rho(d_2)\frac{-1}{X\sigma\sqrt{\tau}}$$

$$S\rho(d_1) = Xe^{-r\tau}\rho(d_2) \Rightarrow \frac{S}{X} = e^{-r\tau}e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2} \Rightarrow$$

$$\Rightarrow \frac{S}{X} = e^{-r\tau}e^{-\frac{\tau\sigma^2}{2}}\exp\left(\frac{\ln S/X\ \sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}\right)\exp\left(\frac{\left(r+\sigma^2/2\right)\tau\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}\right) \Rightarrow 1 = 1$$

Result:

$$Dual\ \Delta_{PO} = \frac{\partial P}{\partial X} = e^{-r\tau}N(-d_2)$$

3.6.3. Deduce Relation for Dual Delta Greek with Annual Dividend Yield *d* (Put Option)

Implementation of the basic derivate (with an annual dividend yield *d*)

$$Dual\ \Delta = \frac{\partial P}{\partial X} = -e^{-d\tau}S\frac{\partial N(-d_1)}{\partial d_1}\frac{\partial d_1}{\partial X} + Xe^{-r\tau}\frac{\partial N(-d_2)}{\partial d_2}\frac{\partial d_2}{\partial X} + e^{-r\tau}N(-d_2)$$

Implementation of the partial derivates

$$\frac{\partial N(d_1)}{\partial d_1} = \rho(d_1), \frac{\partial N(d_2)}{\partial d_2} = \rho(d_2)$$

$$\frac{\partial d_1}{\partial X} = \frac{\partial d_2}{\partial X} = \frac{\frac{X}{S} S \frac{-1}{X^2}}{\sigma\sqrt{\tau}} = \frac{-1}{X\sigma\sqrt{\tau}}$$

Adjustment of the basic derivate (with an annual dividend yield *d*)

$$e^{-d\tau} S \frac{\partial N(-d_1)}{\partial d_1} \frac{\partial d_1}{\partial X} = -e^{-d\tau} S \rho(d_1) \frac{-1}{X\sigma\sqrt{\tau}},$$

$$Xe^{-r\tau} \frac{\partial N(-d_2)}{\partial d_2} \frac{\partial d_2}{\partial X} = -Xe^{-r\tau} \rho(d_2) \frac{-1}{X\sigma\sqrt{\tau}}$$

$$e^{-d\tau} S \rho(d_1) = Xe^{-r\tau} \rho(d_2) \Rightarrow e^{-d\tau} \frac{S}{X} = e^{-r\tau} e^{d_1\sigma\sqrt{\tau}} e^{-\tau\sigma^2} \Rightarrow$$

$$\Rightarrow e^{-d\tau} \frac{S}{X} = e^{-r\tau} e^{-\frac{\tau\sigma^2}{2}} \exp\left(\frac{\ln S/X \, \sigma\sqrt{\tau}}{\sigma\sqrt{\tau}} \right) \exp\left(\frac{\left(r - d + \sigma^2/2 \right) \tau\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}} \right) \Rightarrow 1 = 1$$

Result:

$$Dual\ \Delta_{PO} = \frac{\partial P}{\partial X} = e^{-r\tau} N(-d_2)$$

4. Sense of Delta Greek (DvalueDspot), Dual Delta Greek (DvalueDstrike)

(quoted according to http://en.wikipedia.org/wiki/Greeks_(finance))

4.1. Practical Use of Delta Greek

Even though delta will be a number between 0.0 and 1.0 for a long call (and/or short put) and 0.0 and -1.0 for a long put (and/or short call), these numbers are commonly presented as a percentage of the total number of shares represented by the option contract(s). This is convenient because the option will (instantaneously) behave like the number of shares indicated by the delta. For example, if an American call option on XYZ has a delta of 0.25, it will gain or lose value just like 25% of 100 shares or 25 shares of XYZ as the price changes for small price movements.

4.2. As a Proxy for Probability

Some option traders also use the absolute value of delta as the probability that the option will expire in-the-money (if the market moves under Brownian motion). For example, if an out-of-the-money call option has a delta of 0.15, the trader might estimate that the option has appropriately a 15% chance of expiring in-the-money. Similarly, if a put contract has a delta of -0.25, the trader might expect the option to have a 25% probability of expiring in-the-money. At-the-money puts and calls have a delta of approximately 0.5 and -0.5 respectively (however, this approximation rapidly goes out the window when looking at a term of just a few years, with the ATM call commonly having a delta over .60 or .70), or each will have a 50% chance of expiring in-the-money. The correct, exact calculation for the probability of an option finishing in the money is its Dual Delta, which is the first derivative of option price with respect to strike.

4.3. Relationship between Call and Put Delta

Given a call and put option for the same underlying, strike price and time to maturity, the sum of the absolute values of the delta of each option will be 1.00

If the value of delta for an option is known, one can compute the value of the option of the same strike price, underlying and maturity but opposite right by subtracting 1 from the known value. For example, if the delta of a call is .42 then one can compute the delta of the corresponding put at the same strike price by 0.42 - 1 = -0.58.

4.4. Practical Use of Dual Delta Greek

Although Dual Delta Greek is a primary input into the Black-Scholes model, the overall impact on the value function of an option corresponding to changes in the strike price is generally insignificant and therefore higher-order derivates involving the strike price are not common.

5. Conclusion

The formulas for Delta Greek and Dual Delta Greek were derived through the medium of mathematical data mining tools in statistics education – through the medium of both the differential calculus and the fundamental theorem of integral calculus (see Záškodný,P., Havlíček,I., Budinský,P. (2010-2011)).

Formulas for Delta Greek Δ

$$\Delta_{CO} = e^{-d\tau} N\left(d_1\right)$$

$$\Delta_{PO} = -e^{-d\tau} N\left(-d_1\right)$$

Formulas for Dual Delta Greek Dual Δ

$$\text{Dual } \Delta_{CO} = -e^{-r\tau} N\left(d_2\right)$$

$$\text{Dual } \Delta_{PO} = e^{-r\tau} N\left(-d_2\right)$$

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ISBN 978-80-89160-78-5

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- http://en.wikipedia.org/wiki/Greeks_(finance)

Application of Mathematical Data Mining Tools – Greeks of First Order (Theta, Rho, Vega)

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Abstract:

An imperative of data mining and a need of cooperation of the human with today´s computers are emphasized by D.A.Keim (Keim, 2002):

"The progress made in hardware technology allows today´s computer systems to store very large amounts of data. Researchers from the University of Berkeley estimate that every year 1 Exabyte (= 1 Million Terabyte) of data are generated, of which a large portion is available in digital form. This means that in the next three years more data will be generated than in all of human history before".

"If the data is presented textually, the amount of data which can be displayed is in range one hundred data items, but this is like a drop in the ocean when dealing with data sets containing millions of data items".

"For data mining to be effective, it is important to include the human in the data exploration process and combine the flexibility, creativity, and general knowledge of the human with the enormous storage capacity and the computational power of today´s computers."

The basic concepts about Data Mining in Statistics Education – see Záškodný,P., Tarábek,P. (2010-2011), ***Data Mining Tools in Statistics Education*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.) The complex data mining tool in statistics education is the Curricular Process of Statistics. The partial data mining tools in statistics education are the Analytical Synthetic Modelling, Statistical and Mathematical Data Mining Tools.

Description of Statistical and Mathematical Data Mining Tools in Statistics Education – see Záškodný,P., Havlíček,I., Budinský,P. (2010-2011), ***Partial Data Mining Tools in Statistics Education – in Greeks and Option Hedging*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.)

Description of Greeks of First Order in Statistics Education – see Záškodný,P., Havlíček,I., Budinský,P., Hrdlička,L. (2010-2011), ***Where will be used the partial data mining tools in statistics education? In Greeks*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.)

The application of mathematical data mining tools in the light of a deduction of the first order greeks "Theta", "Rho", "Vega" (based on the Black-Scholes Model) will be described in this paper.

The main principle of paper: Mathematical Data Mining Tools in Statistics Education
The main goal of paper: Deduction of Greeks of First Order
The procedure of paper: Survey of Greeks of First order
Deduction of Theta Greek
Deduction of Rho Greek
Deduction of Vega Greek
Exercising Theta Greek
Exercising Rho Greek
Exercising Vega Greek

The results of paper:

1. *Basic Mathematical Derivation of Theta Greek*
2. *Basic Mathematical Derivation of Rho Greek*
3. *Basic Mathematical Derivation of Vega Greek*
4. *Partial Mathematical Derivation of Theta Greek*
5. *Partial Mathematical Derivation of Rho Greek*
6. *Partial Mathematical Derivation of Vega Greek*

Key Words:

Data Mining, Statistics Education, Partial Data Mining Tool – Mathematical Data Mining Tools, Black-Scholes Model, Value Function, Greeks of First Order, Theta Greek, Rho Greek, Vega Greek, Derivation of Formulas for First Order Greeks “Theta”, “Rho”, “Vega”

1. Survey of Greeks of First Order

The Black-Scholes model traces the evolution of the option´s key underlying variables in continuous-time. This is done by means of both the standard normal probability densities $\rho(d_1)$, $\rho(d_2)$ and the standard normal distribution functions $N(d_1)$, $N(d_2)$.

The variables d_1, d_2 are connected with Spot price S, Strike price X, Risk-Free Rate r, Annual Dividend Yield d, Time to Maturity τ, and Volatility σ.

Value Function *V* (as Fair Price or as Premium) can be expressed as a function of five quantities $V = \mathrm{f}\,(S, X, r, \tau, \sigma)$

Survey of greeks of the first order as survey of the speeds of value function change (see Záškodný,P., Havlíček,I., Budinský,P., Hrdlička,L. (2010-2011):

$$\Delta = \frac{\partial V}{\partial S} = \text{DvalueDspot = Delta Greek}$$

$$\text{Dual}\,\Delta = \frac{\partial V}{\partial X} = \text{DvalueDstrike = Dual Delta Greek}$$

$$\Theta = -\frac{\partial V}{\partial \tau} = -\text{DvalueDtime = Theta Greek}$$

$$\rho = \frac{\partial V}{\partial r} = \text{DvalueDrate = Rho Greek}$$

$$\nu\left(\text{Vega}\right) = \frac{\partial V}{\partial \sigma} = \text{DvalueDvol = Vega Greek}$$

2. Mathematical Derivation of Theta Greek (DvalueDtime)

2.1. Basic Relations

$$\Theta = -\frac{\partial V}{\partial \tau}, V = \langle C \rangle, V = \langle P \rangle$$
$$\langle C \rangle = Se^{-d\tau} N(d_1) - Xe^{-r\tau} N(d_2), \langle P \rangle = Xe^{-r\tau} N(-d_2) - Se^{-d\tau} N(-d_1)$$
$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}$$
$$N(d_1) = \int_{-\infty}^{d_1} \rho(d_1) d(d_1), N(d_2) = \int_{-\infty}^{d_2} \rho(d_2) d(d_2)$$
$$N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$$
$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}, \rho(d_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}}, \rho(d_2) = \rho(d_1) e^{d_1\sigma\sqrt{\tau}} e^{-\tau\sigma^2/2}$$
$$e^{d_1\sigma\sqrt{\tau}} = \frac{S}{X} e^{\left(r+\sigma^2/2\right)\tau} e^{-d\tau}$$
$$e^{-d_2\sigma\sqrt{\tau}} = \frac{S}{X} e^{-\left(r-\sigma^2/2\right)\tau} e^{d\tau}$$

2.2. Implementation of Basic Derivate (without Annual Dividend Yield *d*)

$$\Theta = -\frac{\partial C}{\partial \tau} = -S \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial \tau} + Xe^{-r\tau} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial \tau} - Xre^{-r\tau} N(d_2)$$

2.3. Implementation of Partial Derivates

$$\frac{\partial N(d_1)}{\partial d_1} = \rho(d_1), \frac{\partial N(d_2)}{\partial d_2} = \rho(d_2)$$
$$\frac{\partial d_2}{\partial \tau} = \frac{\partial d_1}{\partial \tau} - \frac{\sigma}{2\sqrt{\tau}}$$

2.4. Adjustment of Implemented Basic Derivate (without Annual Dividend Yield *d*)

$$-S\rho(d_1)\frac{\partial d_1}{\partial \tau} + Xe^{-r\tau}\rho(d_2)\frac{\partial d_2}{\partial \tau} = -S\rho(d_1)\frac{\partial d_1}{\partial \tau} + Xe^{-r\tau}\rho(d_1) e^{d_1\sigma\sqrt{\tau}} e^{-\tau\sigma^2/2} \frac{\partial d_2}{\partial \tau}$$

On the basis of relation $e^{d_1\sigma\sqrt{\tau}} = \frac{S}{X} e^{\left(r+\sigma^2/2\right)\tau}$ from paragraph 2.1., it is possible to obtain

$$-S\rho(d_1)\frac{\partial d_1}{\partial\tau}+Xe^{-r\tau}\rho(d_1)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2}\frac{\partial d_2}{\partial\tau}\Rightarrow$$
$$-S\rho(d_1)\frac{\partial d_1}{\partial\tau}+Xe^{-r\tau}\rho(d_1)\frac{S}{X}e^{(r+\sigma^2/2)\tau}e^{-\tau\sigma^2/2}\frac{\partial d_2}{\partial\tau}\Rightarrow$$
$$-S\rho(d_1)\frac{\partial d_1}{\partial\tau}+S\rho(d_1)\frac{\partial d_2}{\partial\tau}=S\rho(d_1)\left(\frac{\partial d_2}{\partial\tau}-\frac{\partial d_1}{\partial\tau}\right)$$

On the basis of relation $\frac{\partial d_2}{\partial\tau}=\frac{\partial d_1}{\partial\tau}-\frac{\sigma}{2\sqrt{\tau}}$ from paragraph 2.3., it is possible to obtain

$$S\rho(d_1)\left(\frac{\partial d_2}{\partial\tau}-\frac{\partial d_1}{\partial\tau}\right)=-\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}}$$

2.5. Summary

Formulas for Theta Greek (Call Option without Annual Dividend Yield *d* and with Annual Dividend Yield *d*):

$$\Theta=-\frac{\partial C}{\partial\tau}=-\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}}-rXe^{-r\tau}N(d_2)$$
$$\Theta=-\frac{\partial C}{\partial\tau}=-e^{-d\tau}\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}}-rXe^{-r\tau}N(d_2)$$

Formulas for Theta Greek (Put Option without Annual Dividend Yield *d* and with Annual Dividend Yield *d*, and Relations $N(d_1)+N(-d_1)=1$, $N(d_2)+N(-d_2)=1$):

$$\Theta=-\frac{\partial P}{\partial\tau}=-\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}}+rXe^{-r\tau}N(-d_2)$$
$$\Theta=-\frac{\partial P}{\partial\tau}=-e^{-d\tau}\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}}+rXe^{-r\tau}N(-d_2)$$

2.6. Exercising on Derivation of Theta Greek

2.6.1. Deduce Relation for Theta Greek with Annual Dividend Yield *d* (Call Option)

Implementation of a basic derivate (with an annual dividend yield *d*)

$$\Theta=-\frac{\partial C}{\partial\tau}=-e^{-d\tau}S\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial\tau}+Xe^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial\tau}-Xre^{-r\tau}N(d_2)$$

Implementation of the partial derivates

$$\frac{\partial N(d_1)}{\partial d_1}=\rho(d_1),\ \frac{\partial N(d_2)}{\partial d_2}=\rho(d_2)$$
$$\frac{\partial d_2}{\partial\tau}=\frac{\partial d_1}{\partial\tau}-\frac{\sigma}{2\sqrt{\tau}}$$

Adjustment of the basic derivate (with an annual dividend yield *d*)

$$-e^{-d\tau}S\rho(d_1)\frac{\partial d_1}{\partial\tau}+Xe^{-r\tau}\rho(d_2)\frac{\partial d_2}{\partial\tau}=-e^{-d\tau}S\rho(d_1)\frac{\partial d_1}{\partial\tau}+Xe^{-r\tau}\rho(d_1)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2}\frac{\partial d_2}{\partial\tau}$$

On the basis of relation $e^{d_1\sigma\sqrt{\tau}}=\frac{S}{X}e^{(r+\sigma^2/2)\tau}e^{-d\tau}$ from paragraph 2.1., it is possible to obtain

$$-e^{-d\tau}S\rho(d_1)\frac{\partial d_1}{\partial\tau}+Xe^{-r\tau}\rho(d_1)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2}\frac{\partial d_2}{\partial\tau}\Rightarrow$$

$$-e^{-d\tau}S\rho(d_1)\frac{\partial d_1}{\partial\tau}+Xe^{-r\tau}\rho(d_1)\frac{S}{X}e^{(r+\sigma^2/2)\tau}e^{-d\tau}e^{-\tau\sigma^2/2}\frac{\partial d_2}{\partial\tau}\Rightarrow$$

$$-e^{-d\tau}S\rho(d_1)\frac{\partial d_1}{\partial\tau}+e^{-d\tau}S\rho(d_1)\frac{\partial d_2}{\partial\tau}=e^{-d\tau}S\rho(d_1)\left(\frac{\partial d_2}{\partial\tau}-\frac{\partial d_1}{\partial\tau}\right)$$

On the basis of relation $\frac{\partial d_2}{\partial\tau}=\frac{\partial d_1}{\partial\tau}-\frac{\sigma}{2\sqrt{\tau}}$ from paragraph 2.3., it is possible to obtain

$$e^{-d\tau}S\rho(d_1)\left(\frac{\partial d_2}{\partial\tau}-\frac{\partial d_1}{\partial\tau}\right)=-\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}}e^{-d\tau}$$

Result:

$$\Theta=-\frac{\partial C}{\partial\tau}=-e^{-d\tau}\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}}-rXe^{-r\tau}N(d_2)$$

2.6.2. Deduce Relation for Theta Greek without Annual Dividend Yield *d* (Put Option)

Implementation of a basic derivate (without an annual dividend yield *d*)

$$\Theta=-\frac{\partial P}{\partial\tau}=S\frac{\partial N(-d_1)}{\partial d_1}\frac{\partial d_1}{\partial\tau}-Xe^{-r\tau}\frac{\partial N(-d_2)}{\partial d_2}\frac{\partial d_2}{\partial\tau}+Xre^{-r\tau}N(-d_2)$$

$$N(d_1)+N(-d_1)=1,\ N(d_2)+N(-d_2)=1$$

$$\Theta=-\frac{\partial P}{\partial\tau}=-S\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial\tau}+Xe^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial\tau}+Xre^{-r\tau}N(-d_2)$$

Implementation of the partial derivates

$$\frac{\partial N(d_1)}{\partial d_1}=\rho(d_1),\ \frac{\partial N(d_2)}{\partial d_2}=\rho(d_2)$$

$$\frac{\partial d_2}{\partial\tau}=\frac{\partial d_1}{\partial\tau}-\frac{\sigma}{2\sqrt{\tau}}$$

Adjustment of the basic derivate (without an annual dividend yield *d*) is identical with the procedure implemented in paragraph 2.4.

Result:

$$\Theta=-\frac{\partial P}{\partial\tau}=-\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}}-rXe^{-r\tau}N(-d_2)$$

2.6.3. Deduce Relation for Theta Greek with Annual Dividend Yield *d* (Put Option)

Implementation of a basic derivate (with an annual dividend yield *d*)

$$\Theta = -\frac{\partial P}{\partial \tau} = e^{-d\tau} S \frac{\partial N(-d_1)}{\partial d_1}\frac{\partial d_1}{\partial \tau} - Xe^{-r\tau}\frac{\partial N(-d_2)}{\partial d_2}\frac{\partial d_2}{\partial \tau} + Xre^{-r\tau}N(-d_2)$$

$$N(d_1)+N(-d_1)=1,\ N(d_2)+N(-d_2)=1$$

$$\Theta = -\frac{\partial P}{\partial \tau} = -e^{-d\tau} S \frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \tau} + Xe^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial \tau} + Xre^{-r\tau}N(-d_2)$$

Implementation of the partial derivates

$$\frac{\partial N(d_1)}{\partial d_1} = \rho(d_1),\ \frac{\partial N(d_2)}{\partial d_2} = \rho(d_2)$$

$$\frac{\partial d_2}{\partial \tau} = \frac{\partial d_1}{\partial \tau} - \frac{\sigma}{2\sqrt{\tau}}$$

Adjustment of the basic derivate (with an annual dividend yield *d*) is identical with the procedure implemented in paragraph 2.4.

Result:

$$\Theta = -\frac{\partial P}{\partial \tau} = -e^{-d\tau}\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}} - rXe^{-r\tau}N(-d_2)$$

3. Mathematical Derivation of Rho Greek (DvalueDrate)

3.1. Basic Relations

$$\rho = \frac{\partial V}{\partial r},\ V = \langle C\rangle,\ V = \langle P\rangle$$

$$\langle C\rangle = Se^{-d\tau}N(d_1) - Xe^{-r\tau}N(d_2),\ \langle P\rangle = Xe^{-r\tau}N(-d_2) - Se^{-d\tau}N(-d_1)$$

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}},\ d_2 = d_1 - \sigma\sqrt{\tau}$$

$$N(d_1) = \int_{-\infty}^{d_1}\rho(d_1)\,d(d_1),\ N(d_2) = \int_{-\infty}^{d_2}\rho(d_2)\,d(d_2)$$

$$N(d_1)+N(-d_1)=1,\ N(d_2)+N(-d_2)=1$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}},\ \rho(d_2) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_2^2}{2}},\ \rho(d_2) = \rho(d_1)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2}$$

$$e^{d_1\sigma\sqrt{\tau}} = \frac{S}{X}e^{\left(r+\sigma^2/2\right)\tau}e^{-d\tau}$$

$$e^{-d_2\sigma\sqrt{\tau}} = \frac{S}{X}e^{-\left(r-\sigma^2/2\right)\tau}e^{d\tau}$$

3.2. Implementation of Basic Derivate (without Annual Dividend Yield *d*)

$$\rho = \frac{\partial C}{\partial r} = S\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial r} - Xe^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial r} + X\tau e^{-r\tau}N(d_2)$$

3.3. Implementation of Partial Derivates

$$\frac{\partial N(d_1)}{\partial d_1} = \rho(d_1), \frac{\partial N(d_2)}{\partial d_2} = \rho(d_2)$$
$$\frac{\partial d_2}{\partial r} = \frac{\partial d_1}{\partial r} = \frac{\tau}{\sigma\sqrt{\tau}}$$

3.4. Adjustment of Implemented Basic Derivate (without Annual Dividend Yield *d*)

$$\rho = \frac{\partial C}{\partial r} = S\rho(d_1)\frac{\partial d_1}{\partial r} - Xe^{-r\tau}\rho(d_2)\frac{\partial d_2}{\partial r} + X\tau e^{-r\tau}N(d_2)$$
$$\rho = \frac{\partial C}{\partial r} = S\rho(d_1)\frac{\tau}{\sigma\sqrt{\tau}} - Xe^{-r\tau}\rho(d_2)\frac{\tau}{\sigma\sqrt{\tau}} + X\tau e^{-r\tau}N(d_2)$$

On the basis of relation $\rho(d_2) = \rho(d_1)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2}$ (see paragraph 3.1.), the equality

$$S\rho(d_1)\frac{\tau}{\sigma\sqrt{\tau}} = Xe^{-r\tau}\rho(d_2)\frac{\tau}{\sigma\sqrt{\tau}}$$

to be proved:

$$S\rho(d_1) = Xe^{-r\tau}\rho(d_1)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2} \Rightarrow$$
$$\frac{S}{X} = e^{-r\tau}e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2} = e^{-(r+\sigma^2/2)\tau}e^{d_1\sigma\sqrt{\tau}}$$

By the installment $d_1 = \frac{\ln S/X + \left(r + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$ (see paragraph 3.1.), it is possible to obtain wanted equality

$$\frac{S}{X} = e^{-(r+\sigma^2/2)\tau}e^{d_1\sigma\sqrt{\tau}} = e^{-(r+\sigma^2/2)\tau}\frac{S}{X}e^{(r+\sigma^2/2)\tau} \Rightarrow 1 = 1$$

3.5. Summary

Formulas for Rho Greek (Call Option without Annual Dividend Yield *d* and with Annual Dividend Yield *d*):

$$\rho = \frac{\partial C}{\partial r} = \tau Xe^{-r\tau}N(d_2)$$

Formulas for Rho Greek (Put Option without Annual Dividend Yield *d* and with Annual Dividend Yield *d*, and Relations $N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$):

$$\rho = \frac{\partial P}{\partial r} = -\tau Xe^{-r\tau}N(-d_2)$$

3.6. Exercising on Derivation of Rho Greek

3.6.1. Deduce Relation for Rho Greek with Annual Dividend Yield *d* (Call Option)

The adjustment of basic derivate, implemented in paragraph 3.2., and completed by implemented partial derivates in paragraph 3.3., will be supplemented by an annual dividend yield *d*, i.e. to be used the relations from paragraph 3.1. in the shape

$$\langle C\rangle = Se^{-d\tau} N\left(d_1\right) - Xe^{-r\tau} N\left(d_2\right),\ d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}.$$

Hence it follows the procedure (see paragraph 3.4.)

$$\rho = \frac{\partial C}{\partial r} = e^{-d\tau} S\rho\left(d_1\right)\frac{\partial d_1}{\partial r} - Xe^{-r\tau}\rho\left(d_2\right)\frac{\partial d_2}{\partial r} + X\tau e^{-r\tau} N\left(d_2\right)$$
$$\rho = \frac{\partial C}{\partial r} = e^{-d\tau} S\rho\left(d_1\right)\frac{\tau}{\sigma\sqrt{\tau}} - Xe^{-r\tau}\rho\left(d_2\right)\frac{\tau}{\sigma\sqrt{\tau}} + X\tau e^{-r\tau} N\left(d_2\right)$$

and again through the medium of relation $\rho\left(d_2\right) = \rho\left(d_1\right)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2}$ (see paragraph 3.1.) the equality

$$e^{-d\tau} S\rho\left(d_1\right)\frac{\tau}{\sigma\sqrt{\tau}} = Xe^{-r\tau}\rho\left(d_2\right)\frac{\tau}{\sigma\sqrt{\tau}}$$

to be proved:

$$e^{-d\tau} S\rho\left(d_1\right) = Xe^{-r\tau}\rho\left(d_1\right)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2} \Rightarrow$$
$$e^{-d\tau}\frac{S}{X} = e^{-r\tau}e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2} = e^{-\left(r+\sigma^2/2\right)\tau}e^{d_1\sigma\sqrt{\tau}}$$
$$e^{-d\tau}\frac{S}{X} = e^{-\left(r+\sigma^2/2\right)\tau}e^{d_1\sigma\sqrt{\tau}} = e^{-\left(r+\sigma^2/2\right)\tau}e^{-d\tau}\frac{S}{X}e^{\left(r+\sigma^2/2\right)\tau} \Rightarrow 1 = 1$$

Result:

$$\rho = \frac{\partial C}{\partial r} = \tau Xe^{-r\tau} N\left(d_2\right)$$

3.6.2. Deduce Relation for Rho Greek without Annual Dividend Yield *d* (Put Option)

Implementation of a basic derivate (without an annual dividend yield *d*)

$$\rho = \frac{\partial P}{\partial r} = -S\frac{\partial N\left(-d_1\right)}{\partial d_1}\frac{\partial d_1}{\partial r} + Xe^{-r\tau}\frac{\partial N\left(-d_2\right)}{\partial d_2}\frac{\partial d_2}{\partial r} - X\tau e^{-r\tau} N\left(-d_2\right)$$

Implementation of the partial derivates

$$\frac{\partial N(d_1)}{\partial d_1} = \rho(d_1), \frac{\partial N(d_2)}{\partial d_2} = \rho(d_2)$$

$$\frac{\partial d_2}{\partial r} = \frac{\partial d_1}{\partial r} = \frac{\tau}{\sigma\sqrt{\tau}}$$

Adjustment of implemented the basic derivate (without an annual dividend yield *d*) by means of the relations $N(d_1)+N(-d_1)=1, N(d_2)+N(-d_2)=1$ from paragraph 3.1.

$$\rho = \frac{\partial P}{\partial r} = -S\frac{\partial(1-N(d_1))}{\partial d_1}\frac{\partial d_1}{\partial r} + Xe^{-r\tau}\frac{\partial(1-N(d_2))}{\partial d_2}\frac{\partial d_2}{\partial r} - X\tau e^{-r\tau}N(-d_2)$$

$$\rho = \frac{\partial P}{\partial r} = S\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial r} - Xe^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial r} - X\tau e^{-r\tau}N(-d_2)$$

$$\rho = \frac{\partial P}{\partial r} = S\rho(d_1)\frac{\partial d_1}{\partial r} - Xe^{-r\tau}\rho(d_2)\frac{\partial d_2}{\partial r} - X\tau e^{-r\tau}N(-d_2)$$

$$\rho = \frac{\partial P}{\partial r} = S\rho(d_1)\frac{\tau}{\sigma\sqrt{\tau}} - Xe^{-r\tau}\rho(d_2)\frac{\tau}{\sigma\sqrt{\tau}} - X\tau e^{-r\tau}N(-d_2)$$

Next procedure is identical with the procedure in paragraph 3.4. Through the medium of relations

$$\rho(d_2) = \rho(d_1)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2}, \; d_1 = \frac{\ln S/X + \left(r + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$ (see paragraph 3.1.)

the equality

$$S\rho(d_1)\frac{\tau}{\sigma\sqrt{\tau}} = Xe^{-r\tau}\rho(d_2)\frac{\tau}{\sigma\sqrt{\tau}}$$

to be proved by the procedure identical with the procedure in paragraph 3.6.1.

Result:

$$\rho = \frac{\partial P}{\partial r} = -rXe^{-r\tau}N(-d_2)$$

3.6.3. Deduce Relation for Rho Greek with Annual Dividend Yield *d* (Put Option)

Implementation of a basic derivate (with an annual dividend yield *d*)

$$\rho = \frac{\partial P}{\partial r} = -e^{-d\tau} S \frac{\partial N(-d_1)}{\partial d_1} \frac{\partial d_1}{\partial r} + Xe^{-r\tau} \frac{\partial N(-d_2)}{\partial d_2} \frac{\partial d_2}{\partial r} - X\tau e^{-r\tau} N(-d_2)$$

Implementation of the partial derivates

$$\frac{\partial N(d_1)}{\partial d_1} = \rho(d_1), \frac{\partial N(d_2)}{\partial d_2} = \rho(d_2)$$

$$\frac{\partial d_2}{\partial r} = \frac{\partial d_1}{\partial r} = \frac{\tau}{\sigma\sqrt{\tau}}$$

Adjustment of implemented the basic derivate (without an annual dividend yield *d*) by means of the relations $N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$ from paragraph 3.1.

$$\rho = \frac{\partial P}{\partial r} = -e^{-d\tau} S \frac{\partial(1 - N(d_1))}{\partial d_1} \frac{\partial d_1}{\partial r} + Xe^{-r\tau} \frac{\partial(1 - N(d_2))}{\partial d_2} \frac{\partial d_2}{\partial r} - X\tau e^{-r\tau} N(-d_2)$$

$$\rho = \frac{\partial P}{\partial r} = e^{-d\tau} S \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial r} - Xe^{-r\tau} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial r} - X\tau e^{-r\tau} N(-d_2)$$

$$\rho = \frac{\partial P}{\partial r} = e^{-d\tau} S \rho(d_1) \frac{\partial d_1}{\partial r} - Xe^{-r\tau} \rho(d_2) \frac{\partial d_2}{\partial r} - X\tau e^{-r\tau} N(-d_2)$$

$$\rho = \frac{\partial P}{\partial r} = e^{-d\tau} S \rho(d_1) \frac{\tau}{\sigma\sqrt{\tau}} - Xe^{-r\tau} \rho(d_2) \frac{\tau}{\sigma\sqrt{\tau}} - X\tau e^{-r\tau} N(-d_2)$$

Next procedure is identical with the procedure in paragraph 3.6.1. By means of the relations

$$\rho(d_2) = \rho(d_1) e^{d_1 \sigma\sqrt{\tau}} e^{-\tau\sigma^2/2}, \ d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$ (see paragraph 3.1.)

the equality

$$e^{-d\tau} S \rho(d_1) \frac{\tau}{\sigma\sqrt{\tau}} = Xe^{-r\tau} \rho(d_2) \frac{\tau}{\sigma\sqrt{\tau}}$$

to be proved.

Result:

$$\rho = \frac{\partial P}{\partial r} = -rXe^{-r\tau} N(-d_2)$$

4. Mathematical Derivation of Vega Greek (DvalueDvol)

4.1. Basic Relations

$$\nu = \frac{\partial V}{\partial \sigma}, V = \langle C \rangle, V = \langle P \rangle$$

$$\langle C \rangle = Se^{-d\tau} N(d_1) - Xe^{-r\tau} N(d_2), \langle P \rangle = Xe^{-r\tau} N(-d_2) - Se^{-d\tau} N(-d_1)$$

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}$$

$$N(d_1) = \int_{-\infty}^{d_1} \rho(d_1)\, d(d_1), N(d_2) = \int_{-\infty}^{d_2} \rho(d_2)\, d(d_2)$$

$$N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}, \rho(d_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}}, \rho(d_2) = \rho(d_1) e^{d_1\sigma\sqrt{\tau}} e^{-\tau\sigma^2/2}$$

$$e^{d_1\sigma\sqrt{\tau}} = \frac{S}{X} e^{\left(r+\sigma^2/2\right)\tau} e^{-d\tau}$$

$$e^{-d_2\sigma\sqrt{\tau}} = \frac{S}{X} e^{-\left(r-\sigma^2/2\right)\tau} e^{d\tau}$$

4.2. Implementation of Basic Derivate (without Annual Dividend Yield *d*)

$$\nu = \frac{\partial C}{\partial \sigma} = S \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial \sigma} - Xe^{-r\tau} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial \sigma}$$

4.3. Implementation of Partial Derivates

$$\frac{\partial N(d_1)}{\partial d_1} = \rho(d_1), \frac{\partial N(d_2)}{\partial d_2} = \rho(d_2)$$

$$\frac{\partial d_2}{\partial S} = \frac{\partial d_1}{\partial S} - \sqrt{\tau}$$

4.4. Adjustment of Implemented Basic Derivate (without Annual Dividend Yield *d*)

$$\nu = \frac{\partial C}{\partial \sigma} = S\rho(d_1)\frac{\partial d_1}{\partial \sigma} - Xe^{-r\tau}\rho(d_2)\left(\frac{\partial d_1}{\partial \sigma} - \sqrt{\tau}\right)$$

$$\nu = S\rho(d_1)\frac{\partial d_1}{\partial \sigma} - Xe^{-r\tau}\rho(d_1)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2}\left(\frac{\partial d_1}{\partial \sigma} - \sqrt{\tau}\right)$$

$$\nu = S\rho(d_1)\frac{\partial d_1}{\partial \sigma} - Xe^{-r\tau}\rho(d_1)\frac{S}{X}e^{\left(r+\sigma^2/2\right)\tau}e^{-\tau\sigma^2/2}\left(\frac{\partial d_1}{\partial \sigma} - \sqrt{\tau}\right)$$

$$\nu = S\rho(d_1)\frac{\partial d_1}{\partial \sigma} - X\rho(d_1)\frac{S}{X}\left(\frac{\partial d_1}{\partial \sigma} - \sqrt{\tau}\right)$$

$$\nu = S\rho(d_1)\frac{\partial d_1}{\partial \sigma} - S\rho(d_1)\frac{\partial d_1}{\partial \sigma} + S\rho(d_1)\sqrt{\tau}$$

4.5. Summary

Formulas for Vega Greek (Call Option without Annual Dividend Yield *d* and with Annual Dividend Yield *d*):

$$\nu = \frac{\partial C}{\partial \sigma} = S\ \rho(d_1)\sqrt{\tau}, \nu = \frac{\partial C}{\partial \sigma} = e^{-d\tau} S\ \rho(d_1)\sqrt{\tau}$$
$$\nu = \frac{\partial C}{\partial \sigma} = Xe^{-r\tau}\rho(d_2)\sqrt{\tau}, \nu = \frac{\partial C}{\partial \sigma} = Xe^{-r\tau}\rho(d_2)\sqrt{\tau}$$

Formulas for Vega Greek (Put Option without Annual Dividend Yield *d* and with Annual Dividend Yield *d*, and Relations $N(d_1)+N(-d_1)=1,\ N(d_2)+N(-d_2)=1$):

$$\nu = \frac{\partial P}{\partial \sigma} = S\ \rho(d_1)\sqrt{\tau}, \nu = \frac{\partial P}{\partial \sigma} = e^{-d\tau} S\ \rho(d_1)\sqrt{\tau}$$

4.6. Exercising on Derivation of Vega Greek

4.6.1. Deduce Relation for Vega Greek Using Variable d_2 (Call Option)

Takeover of the needful relations and their adjustment

$$\nu = \frac{\partial C}{\partial \sigma} = S\ \rho(d_1)\sqrt{\tau}, \nu = \frac{\partial C}{\partial \sigma} = e^{-d\tau} S\ \rho(d_1)\sqrt{\tau}$$
(see paragraph 4.5.)

$$\rho(d_2) = \rho(d_1)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2} \Rightarrow \rho(d_1) = \rho(d_2)e^{-d_1\sigma\sqrt{\tau}}e^{\tau\sigma^2/2}$$
(see paragraph 4.1. and elementary adjustment)

$$e^{d_1\sigma\sqrt{\tau}} = \frac{S}{X}e^{(r+\sigma^2/2)\tau}e^{-d\tau} \Rightarrow e^{-d_1\sigma\sqrt{\tau}} = \frac{X}{S}e^{-(r+\sigma^2/2)\tau}e^{d\tau}$$
(see paragraph 4.1. and elementary adjustment)

$$\rho(d_1) = \rho(d_2)e^{-d_1\sigma\sqrt{\tau}}e^{\tau\sigma^2/2} = \rho(d_2)\frac{X}{S}e^{-(r+\sigma^2/2)\tau}e^{d\tau}e^{\tau\sigma^2/2} = \rho(d_2)\frac{X}{S}e^{-r\tau}e^{d\tau}$$
(association of previous adjustments)

Installment of previous adjustments into the relations from paragraph 4.5. for Vega Greek with variable d_1:

$$\nu = \frac{\partial C}{\partial \sigma} = S\ \rho(d_1)\sqrt{\tau} = Xe^{-r\tau}\rho(d_2)\sqrt{\tau},$$
$$\nu = \frac{\partial C}{\partial \sigma} = e^{-d\tau} S\ \rho(d_1)\sqrt{\tau} = e^{d\tau}Xe^{-r\tau}\rho(d_2)\sqrt{\tau}$$

Result (derived formulas for Vega Greek with variable d_2 – call option):

$$\nu = \frac{\partial C}{\partial \sigma} = Xe^{-r\tau}\rho(d_2)\sqrt{\tau}, \nu = \frac{\partial C}{\partial \sigma} = e^{d\tau}Xe^{-r\tau}\rho(d_2)\sqrt{\tau}$$

4.6.2. Deduce Relation for Rho Greek with Annual Dividend Yield *d* (Call Option)

Selection of the needful basic relation from paragraph 4.1.

$$\langle C\rangle = Se^{-d\tau}\,N\left(d_1\right) - Xe^{-r\tau}N\left(d_2\right)$$

Completion of an annual dividend yield *d* into the adjustments implemented in paragraph 4.4.

$$\nu = \frac{\partial C}{\partial\sigma} = e^{-d\tau}S\rho\left(d_1\right)\frac{\partial d_1}{\partial\sigma} - Xe^{-r\tau}\rho\left(d_2\right)\left(\frac{\partial d_1}{\partial\sigma} - \sqrt{\tau}\right)$$
$$\nu = e^{-d\tau}S\rho\left(d_1\right)\frac{\partial d_1}{\partial\sigma} - Xe^{-r\tau}\rho\left(d_1\right)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2}\left(\frac{\partial d_1}{\partial\sigma} - \sqrt{\tau}\right)$$
$$\nu = e^{-d\tau}S\rho\left(d_1\right)\frac{\partial d_1}{\partial\sigma} - Xe^{-r\tau}\rho\left(d_1\right)\frac{S}{X}e^{\left(r-d+\sigma^2/2\right)\tau}e^{-\tau\sigma^2/2}\left(\frac{\partial d_1}{\partial\sigma} - \sqrt{\tau}\right)$$
$$\nu = e^{-d\tau}S\rho\left(d_1\right)\frac{\partial d_1}{\partial\sigma} - e^{-d\tau}X\rho\left(d_1\right)\frac{S}{X}\left(\frac{\partial d_1}{\partial\sigma} - \sqrt{\tau}\right)$$
$$\nu = e^{-d\tau}S\rho\left(d_1\right)\frac{\partial d_1}{\partial\sigma} - e^{-d\tau}S\rho\left(d_1\right)\frac{\partial d_1}{\partial\sigma} + e^{-d\tau}S\rho\left(d_1\right)\sqrt{\tau}$$

Result:

$$\nu = \frac{\partial C}{\partial\sigma} = e^{-d\tau}S\;\rho\left(d_1\right)\sqrt{\tau}$$

4.6.3. Deduce Relation for Vega Greek without Annual Dividend Yield *d* (Put Option)

Selection of the needful basic relations from paragraph 4.1.

$$\langle P\rangle = Xe^{-r\tau}N\left(-d_2\right) - S\,N\left(-d_1\right)$$
$$N\left(d_1\right) + N\left(-d_1\right) = 1,\; N\left(d_2\right) + N\left(-d_2\right) = 1$$

Implementation of a basic derivate (without an annual dividend yield *d*)

$$\nu = \frac{\partial P}{\partial\sigma} = -S\frac{\partial\left(1 - N\left(d_1\right)\right)}{\partial d_1}\frac{\partial d_1}{\partial\sigma} + Xe^{-r\tau}\frac{\partial\left(1 - N\left(d_2\right)\right)}{\partial d_2}\frac{\partial d_2}{\partial\sigma}$$

After adjustment, the relation will be acquired to be identical with the relation from paragraph 4.2.

$$\nu = \frac{\partial P}{\partial\sigma} = S\frac{\partial N\left(d_1\right)}{\partial d_1}\frac{\partial d_1}{\partial\sigma} - Xe^{-r\tau}\frac{\partial N\left(d_2\right)}{\partial d_2}\frac{\partial d_2}{\partial\sigma}$$

Next derivation is identical with paragraphs 4.3. and 4.4. Result:

$$\nu = \frac{\partial P}{\partial\sigma} = S\;\rho\left(d_1\right)\sqrt{\tau}$$

4.6.4. Deduce Relation for Vega Greek with Annual Dividend Yield *d* (Put Option)

Selection of the needful basic relations from paragraph 4.1.

$$\langle P\rangle = Xe^{-r\tau}N\left(-d_2\right)-Se^{-d\tau}\,N\left(-d_1\right)$$
$$N\left(d_1\right)+N\left(-d_1\right)=1,\;N\left(d_2\right)+N\left(-d_2\right)=1$$

Implementation of a basic derivate (with an annual dividend yield *d*)

$$\nu=\frac{\partial P}{\partial\sigma}=-e^{-d\tau}S\frac{\partial\left(1-N\left(d_1\right)\right)}{\partial d_1}\frac{\partial d_1}{\partial\sigma}+Xe^{-r\tau}\frac{\partial\left(1-N\left(d_2\right)\right)}{\partial d_2}\frac{\partial d_2}{\partial\sigma}$$

After adjustment, the relation will be acquired

$$\nu=\frac{\partial P}{\partial\sigma}=e^{-d\tau}S\frac{\partial N\left(d_1\right)}{\partial d_1}\frac{\partial d_1}{\partial\sigma}-Xe^{-r\tau}\frac{\partial N\left(d_2\right)}{\partial d_2}\frac{\partial d_2}{\partial\sigma}$$

This relation is identical with the relation

$$\nu=\frac{\partial C}{\partial\sigma}=e^{-d\tau}S\rho\left(d_1\right)\frac{\partial d_1}{\partial\sigma}-Xe^{-r\tau}\rho\left(d_2\right)\left(\frac{\partial d_1}{\partial\sigma}-\sqrt{\tau}\right),$$

To be acquired in paragraph 4.6.2. – next procedure is adequate to this paragraph

Result:

$$\nu=\frac{\partial P}{\partial\sigma}=e^{-d\tau}S\;\rho\left(d_1\right)\sqrt{\tau}$$

5. Sense of Theta Greek, Rho Greek, Vega Greek

(quoted according to http://en.wikipedia.org/wiki/Greeks_(finance))

5.1. Practical Use of Theta Greek

The mathematical result of the formula for Theta Greek is expressed in value/year. By convention, it is useful to divide the result by the number of days per year to arrive at the amount of money, per share of the underlying that the option loses in one day. Theta Greek is always negative for long calls and puts and positive for short (or written) calls and puts. The Total Theta Greek for a portfolio of options can be determined by simply taking the sum of the thetas for each individual position.

The value of an option is made up of two parts: The intrinsic value and the time value. The intrinsic value is the amount of money you would gain if you exercised the option immediately, so a call with strike \$50 on a stock with price \$60 would have intrinsic value of \$10, whereas the corresponding put would have zero intrinsic value. The time value is the worth of having the option of waiting longer when deciding to exercise. Even a deeply out of the money put will be worth something as there is some chance the stock price will fall below the strike. However, as time approaches maturity, there is less chance of this happening, so the time value of an option is decreasing with time.

Thus if you are long an option you are short theta: your portfolio will lose value with the passage of time (all other factors held constant).

5.2. Practical Use of Rho Greek

Rho is typically expressed as the amount of money, per share, that the value of the option will gain or lose as the rate of return of a risk-free investment rises or falls by 1.0%.

5.3. Practical Use of Vega Greek

Vega is typically expressed as the amount of money, per underlying share the option's value will gain or lose as volatility rises or falls by 1%.

Vega can be an important Greek to monitor for an option trader, especially in volatile markets since some of the value of option strategies can be particularly sensitive to changes in volatility. The value of an option straddle, for example, is extremely dependent on changes to volatility.

6. Conclusion

The formulas for Theta Greek, Rho Greek, and Vega Greek were derived through the medium of mathematical data mining tools in statistics education – through the medium of both the differential calculus and the fundamental theorem of integral calculus (see Záškodný,P., Havlíček,I., Budinský,P. (2010-2011)).

Formulas for Theta Greek Θ

$$\Theta_{CO} = -e^{-d\tau} \frac{S\rho(d_1)\sigma}{2\sqrt{\tau}} - rXe^{-r\tau} N(d_2)$$

$$\Theta_{PO} = -e^{-d\tau} \frac{S\rho(d_1)\sigma}{2\sqrt{\tau}} + rXe^{-r\tau} N(-d_2)$$

Formulas for Rho Greek ρ

$$\rho_{CO} = \tau X e^{-r\tau} N(d_2)$$

$$\rho_{PO} = -\tau X e^{-r\tau} N(-d_2)$$

Formulas for Vega Greek ν

$$\nu_{CO,PO} = e^{-d\tau} S\rho(d_1)\sqrt{\tau} = Xe^{-r\tau}\rho(d_2)\sqrt{\tau}$$

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- http://en.wikipedia.org/wiki/Greeks_(finance)

Application of Mathematical Data Mining Tools – Greeks of Second Order (Gamma,Dual Gamma, Vomma)

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Abstract:

An imperative of data mining and a need of cooperation of the human with today´s computers are emphasized by D.A.Keim (Keim, 2002):

"The progress made in hardware technology allows today´s computer systems to store very large amounts of data. Researchers from the University of Berkeley estimate that every year 1 Exabyte (= 1 Million Terabyte) of data are generated, of which a large portion is available in digital form. This means that in the next three years more data will be generated than in all of human history before".

"If the data is presented textually, the amount of data which can be displayed is in range one hundred data items, but this is like a drop in the ocean when dealing with data sets containing millions of data items".

"For data mining to be effective, it is important to include the human in the data exploration process and combine the flexibility, creativity, and general knowledge of the human with the enormous storage capacity and the computational power of today´s computers."

The basic concepts about Data Mining in Statistics Education – see Záškodný,P., Tarábek,P. (2010-2011), ***Data Mining Tools in Statistics Education*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.) The complex data mining tool in statistics education is the Curricular Process of Statistics. The partial data mining tools in statistics education are the Analytical Synthetic Modelling, Statistical and Mathematical Data Mining Tools.

Description of Statistical and Mathematical Data Mining Tools in Statistics Education – see Záškodný,P., Havlíček,I., Budinský,P. (2010-2011), ***Partial Data Mining Tools in Statistics Education – in Greeks and Option Hedging*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.)

Description of Greeks of Individual Second Order in Statistics Education – see Záškodný,P., Havlíček,I., Budinský,P., Hrdlička,L. (2010-2011), ***Where will be used the partial data mining tools in statistics education? In Greeks*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.)

The application of mathematical data mining tools in the light of a deduction of the individual second order greeks "Gamma", "Dual Gamma", "Vomma" (based on the Black-Scholes Model) will be described in this paper.

The main principle of paper: Mathematical Data Mining Tools in Statistics Education
The main goal of paper: Deduction of Greeks of Individual Second Order
The procedure of paper: Survey of Greeks of Individual Second Order
Deduction of Gamma Greek
Deduction of Dual Gamma Greek
Deduction of Vomma Greek
Exercising Gamma Greek
Exercising Dual Gamma Greek
Exercising Vomma Greek

The results of paper:

1. *Basic Mathematical Derivation of Gamma Greek*
2. *Basic Mathematical Derivation of Dual Gamma Greek*
3. *Basic Mathematical Derivation of Vomma Greek*
4. *Partial Mathematical Derivation of Gamma Greek*
5. *Partial Mathematical Derivation of Dual Gamma Greek*
6. *Partial Mathematical Derivation of Vomma Greek*

Key Words:

Data Mining, Statistics Education, Partial Data Mining Tool – Mathematical Data Mining Tools, Black-Scholes Model, Value Function, Greeks of Individual Second Order, Gamma Greek, Dual Gamma Greek, Vomma Greek, Derivation of Formulas for Individual Second Order Greeks "Gamma", "Dual Gamma", "Vomma"

1. Survey of Greeks of Individual Second Order

The Black-Scholes model traces the evolution of the option´s key underlying variables in continuous-time. This is done by means of both the standard normal probability densities $\rho(d_1)$, $\rho(d_2)$ and the standard normal distribution functions $N(d_1)$, $N(d_2)$.

The variables d_1, d_2 are connected with Spot price S, Strike price X, Risk-Free Rate r, Annual Dividend Yield d, Time to Maturity τ, and Volatility σ.

Value Function *V* (as Fair Price or as Premium) can be expressed as a function of five quantities $V = \mathrm{f}\,(S, X, r, \tau, \sigma)$

Survey of greeks of the individual second order as survey of the accelerations of value function change (see Záškodný,P., Havlíček,I., Budinský,P., Hrdlička,L. (2010-2011):

$$\Gamma = \frac{\partial^2 V}{\partial S^2} = \text{DdeltaDspot} = \text{Gamma}$$

$$\text{Dual}\,\Gamma = \frac{\partial^2 V}{\partial X^2} = \text{DdualdeltaDstrike} = \text{DualGamma}$$

$$\text{Vomma} = \frac{\partial^2 V}{\partial \sigma^2} = \text{DvegaDvol} = \text{Vomma}$$

$$\text{Out of Use} = \frac{\partial^2 V}{\partial \tau^2}$$

$$\text{Out of Use} = \frac{\partial^2 V}{\partial r^2}$$

2. Mathematical Derivation of Gamma Greek (DdeltaDspot)

2.1. Basic Relations

$$\Gamma = \frac{\partial^2 V}{\partial S^2}, V = \langle C \rangle, V = \langle P \rangle$$

$$\langle C \rangle = Se^{-d\tau} N(d_1) - Xe^{-r\tau} N(d_2), \langle P \rangle = Xe^{-r\tau} N(-d_2) - Se^{-d\tau} N(-d_1)$$

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}$$

$$N(d_1) = \int_{-\infty}^{d_1} \rho(d_1)\, d(d_1), N(d_2) = \int_{-\infty}^{d_2} \rho(d_2)\, d(d_2)$$

$$N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}, \rho(d_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}}, \rho(d_2) = \rho(d_1) e^{d_1\sigma\sqrt{\tau}} e^{-\tau\sigma^2/2}$$

2.2. Implementation of Basic Derivate (without Annual Dividend Yield *d*)

$$\Gamma = \frac{\partial^2 C}{\partial S^2} = \frac{\partial N(d_1)}{\partial S} = \frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial S}$$

2.3. Implementation of Partial Derivates

$$\frac{\partial N(d_1)}{\partial d_1} = \rho(d_1)$$

$$\frac{\partial d_1}{\partial S} = \frac{\partial d_2}{\partial S} = \frac{\frac{X}{S}\frac{1}{X}}{\sigma\sqrt{\tau}} = \frac{1}{S\sigma\sqrt{\tau}}$$

2.4. Adjustment of Implemented Basic Derivate (without Annual Dividend Yield *d*)

$$\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial S} = \rho(d_1)\frac{1}{S\sigma\sqrt{\tau}}$$

2.5. Summary

Formulas for Gamma Greek (Both Call Option and Put Option without Annual Dividend Yield *d* and with Annual Dividend Yield *d*):

$$\Gamma = \frac{\partial^2 C}{\partial S^2} = \frac{\partial^2 P}{\partial S^2} = \frac{\rho(d_1)}{S\sigma\sqrt{\tau}}, \Gamma = \frac{\partial^2 C}{\partial S^2} = \frac{\partial^2 P}{\partial S^2} = e^{-d\tau}\frac{\rho(d_1)}{S\sigma\sqrt{\tau}} \ .$$

(See also the relations $N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$.)

2.6. Exercising on Derivation of Gamma Greek

2.6.1. Deduce Relation for Gamma Greek with Annual Dividend Yield *d* (Call Option)

Implementation of a basic derivate (see paragraph 2.2.with an annual dividend yield *d*)

$$\Gamma = \frac{\partial^2 C}{\partial S^2} = e^{-d\tau}\frac{\partial N(d_1)}{\partial S} = e^{-d\tau}\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial S}$$

Adjustment of the basic derivate (see paragraph 2.4. with an annual dividend yield *d*)

$$e^{-d\tau}\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial S} = e^{-d\tau}\rho(d_1)\frac{1}{S\sigma\sqrt{\tau}}$$

Result:

$$\Gamma = \frac{\partial^2 C}{\partial S^2} = e^{-d\tau}\frac{\rho(d_1)}{S\sigma\sqrt{\tau}}$$

2.6.2. Deduce Relation for Gamma Greek without Annual Dividend Yield *d* (Put Option)

Implementation of a basic derivate (see paragraph 2.2.)

$$\Gamma = \frac{\partial^2 P}{\partial S^2} = -\frac{\partial N(-d_1)}{\partial S} = -\frac{\partial\left(1-N(d_1)\right)}{\partial S} = \frac{\partial N(d_1)}{\partial S}$$

Adjustment of the basic derivate (see paragraph 2.4.)

$$\Gamma = \frac{\partial^2 P}{\partial S^2} = \frac{\partial N(d_1)}{\partial S} = \frac{\rho(d_1)}{S\sigma\sqrt{\tau}}$$

Result:

$$\Gamma = \frac{\partial^2 P}{\partial S^2} = \frac{\rho(d_1)}{S\sigma\sqrt{\tau}}$$

2.6.3. Deduce Relation for Gamma Greek with Annual Dividend Yield *d* (Put Option)

With the implementation of paragraphs 2.6.1. and 2.6.2., it is possible gradually to write

$$\Gamma = \frac{\partial^2 P}{\partial S^2} = -e^{-d\tau}\frac{\partial N(-d_1)}{\partial S}, \Gamma = \frac{\partial^2 P}{\partial S^2} = e^{-d\tau}\frac{\partial N(d_1)}{\partial S}.$$

Result:

$$\Gamma = \frac{\partial^2 P}{\partial S^2} = e^{-d\tau}\frac{\rho(d_1)}{S\sigma\sqrt{\tau}} \quad .$$

3. Mathematical Derivation of Dual Gamma Greek (DdualdeltaDstrike)

3.1. Basic Relations

$$Dual\ \Gamma = \frac{\partial^2 V}{\partial X^2}, V = \langle C \rangle, V = \langle P \rangle$$

$$\langle C \rangle = Se^{-d\tau} N(d_1) - Xe^{-r\tau} N(d_2), \langle P \rangle = Xe^{-r\tau} N(-d_2) - Se^{-d\tau} N(-d_1)$$

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}$$

$$N(d_1) = \int_{-\infty}^{d_1} \rho(d_1) d(d_1), N(d_2) = \int_{-\infty}^{d_2} \rho(d_2) d(d_2)$$

$$N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}, \rho(d_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}}, \rho(d_2) = \rho(d_1) e^{d_1\sigma\sqrt{\tau}} e^{-\tau\sigma^2/2}$$

3.2. Implementation of Basic Derivate (without Annual Dividend Yield *d*)

$$Dual\ \Delta_{CO} = \frac{\partial C}{\partial X} = -e^{-r\tau} N(d_2)$$

$$Dual\ \Gamma_{CO} = \frac{\partial^2 C}{\partial X^2} = -e^{-r\tau} \frac{\partial N(d_2)}{\partial X} = -e^{-r\tau} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial X}$$

3.3. Implementation of Partial Derivates

$$\frac{\partial N(d_2)}{\partial d_2} = \rho(d_2)$$

$$\frac{\partial d_1}{\partial X} = \frac{\partial d_2}{\partial X} = \frac{\frac{X}{S} S \frac{-1}{X^2}}{\sigma\sqrt{\tau}} = \frac{-1}{X\sigma\sqrt{\tau}}$$

3.4. Adjustment of Implemented Basic Derivate (without Annual Dividend Yield *d*)

$$\frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial X} = \rho(d_2) \frac{-1}{X\sigma\sqrt{\tau}}$$

3.5. Summary

Formulas for Dual Gamma Greek (Both Call Option and Put Option without Annual Dividend Yield *d* and with Annual Dividend Yield *d*):

$$\text{Dual}\ \Gamma_{CO} = \frac{\partial^2 C}{\partial X^2} = \text{Dual}\ \Gamma_{PO} = \frac{\partial^2 P}{\partial X^2} \Rightarrow \text{Dual}\ \Gamma = \frac{\partial^2 V}{\partial X^2} = e^{-r\tau} \frac{\rho(d_2)}{X\sigma\sqrt{\tau}}$$

(See also the relations $N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$.)

3.6. Exercising on Derivation of Dual Gamma Greek

3.6.1. Deduce Relation for Dual Gamma Greek with Annual Dividend Yield *d* (Call Option)

The expression $e^{-d\tau}$ did not play a role in paragraphs 3.1. to 3.5. The result is identical with/without an annual dividend yield *d*.

Result:

$$\text{Dual } \Gamma = e^{-r\tau} \frac{\rho(d_2)}{X\sigma\sqrt{\tau}}$$

3.6.2. Deduce Relation for Dual Gamma Greek without Annual Dividend Yield *d* (Put Option)

Implementation of a basic derivate (without an annual dividend yield *d*)

$$Dual\ \Delta_{PO} = \frac{\partial P}{\partial X} = e^{-r\tau} N(-d_2)$$

$$Dual\ \Gamma_{PO} = \frac{\partial^2 P}{\partial X^2} = e^{-r\tau} \frac{\partial N(-d_2)}{\partial X} = e^{-r\tau} \frac{\partial N(-d_2)}{\partial d_2} \frac{\partial d_2}{\partial X}$$

Implementation of the partial derivates

$$\frac{\partial N(d_2)}{\partial d_2} = \rho(d_2)$$

$$\frac{\partial d_1}{\partial X} = \frac{\partial d_2}{\partial X} = \frac{\frac{X}{S} S \frac{-1}{X^2}}{\sigma\sqrt{\tau}} = \frac{-1}{X\sigma\sqrt{\tau}}$$

Adjustment of implemented the basic derivate (without an annual dividend yield *d*)

$$N(d_2) + N(-d_2) = 1$$

$$\frac{\partial N(-d_2)}{\partial d_2} \frac{\partial d_2}{\partial X} = -\rho(d_2) \frac{-1}{X\sigma\sqrt{\tau}} = \frac{\rho(d_2)}{X\sigma\sqrt{\tau}}$$

Result:

$$\text{Dual } \Gamma = e^{-r\tau} \frac{\rho(d_2)}{X\sigma\sqrt{\tau}}$$

3.6.3. Deduce Relation for Dual Gamma Greek with Annual Dividend Yield *d* (Put Option)

The expression $e^{-d\tau}$ did not play a role in paragraphs 3.1. to 3.5. The result is identical with/without an annual dividend yield *d*.

Result:

$$\text{Dual }\Gamma = e^{-r\tau}\frac{\rho(d_2)}{X\sigma\sqrt{\tau}}$$

4. Mathematical Derivation of Vomma Greek (DvegaDvol)

4.1. Basic Relations

$$\text{Vomma} = \frac{\partial \nu}{\partial \sigma} = \frac{\partial^2 V}{\partial \sigma^2}, V = \langle C\rangle, V = \langle P\rangle$$

$$\nu = \frac{\partial C}{\partial \sigma} = \frac{\partial P}{\partial \sigma} = S\ \rho(d_1)\sqrt{\tau}, \nu = \frac{\partial C}{\partial \sigma} = \frac{\partial P}{\partial \sigma} = e^{-d\tau}S\ \rho(d_1)\sqrt{\tau}$$

$$\langle C\rangle = Se^{-d\tau}\ N(d_1) - Xe^{-r\tau}N(d_2), \langle P\rangle = Xe^{-r\tau}N(-d_2) - Se^{-d\tau}\ N(-d_1)$$

$$d_1 = \frac{\ln {S}/{X} + \left(r - d + {\sigma^2}/{2}\right)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}$$

$$N(d_1) = \int_{-\infty}^{d_1}\rho(d_1)\,d(d_1), N(d_2) = \int_{-\infty}^{d_2}\rho(d_2)\,d(d_2)$$

$$N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}, \rho(d_2) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_2^2}{2}}, \rho(d_2) = \rho(d_1)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2}$$

$$e^{d_1\sigma\sqrt{\tau}} = \frac{S}{X}e^{\left(r+\sigma^2/2\right)\tau}e^{-d\tau}$$

$$e^{-d_2\sigma\sqrt{\tau}} = \frac{S}{X}e^{-\left(r-\sigma^2/2\right)\tau}e^{d\tau}$$

4.2. Implementation of Basic Derivate (with/without Annual Dividend Yield *d* and for Call/Put Option)

$$\text{Vomma}_{CO,PO} = \frac{\partial \nu}{\partial \sigma} = Se^{-d\tau}\frac{\partial \rho(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \sigma}$$

4.3. Implementation of Partial Derivates

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}} \Rightarrow \frac{\partial \rho(d_1)}{\partial d_1} = -d_1 \rho(d_1)$$

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}} \Rightarrow \frac{\partial d_1}{\partial \sigma} = \frac{-\ln S/X - (r-d)\tau}{\sigma^2\sqrt{\tau}} + \frac{\sqrt{\tau}}{2}$$

4.4. Adjustment of Implemented Basic Derivate

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau} \Rightarrow d_2 = \frac{\ln S/X + \left(r - d - \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$

$$\text{Vomma}_{CO,PO} = \frac{\partial \nu}{\partial \sigma} = Se^{-d\tau} \frac{\partial \rho(d_1)}{\partial d_1} \frac{\partial d_1}{\partial \sigma} = Se^{-d\tau} \rho(d_1)\sqrt{\tau}(-d_1) \frac{-2\ln S/X - 2(r-d)\tau + \sigma^2\tau}{2\sigma^2\sqrt{\tau}}$$

Adjustment of the expression

$$(-d_1)\frac{-2\ln S/X - 2(r-d)\tau + \sigma^2\tau}{2\sigma^2\sqrt{\tau}} = d_1 \frac{\ln S/X + (r-d)\tau - \sigma^2/2\,\tau}{\sigma^2\sqrt{\tau}} = \frac{d_1 d_2}{\sigma}$$

4.5. Summary

Formulas for Vomma Greek (with/without Annual Dividend Yield *d* and for Call/Put Option):

$$Vomma_{CO,PO} = \frac{\partial \nu}{\partial \sigma} = \frac{\partial^2 V}{\partial \sigma^2} = Se^{-d\tau} \rho(d_1)\sqrt{\tau} \frac{d_1 d_2}{\sigma} = \nu \frac{d_1 d_2}{\sigma}, V = \langle C \rangle, V = \langle P \rangle .$$

Without an annual dividend yield, the member $e^{-d\tau}$ will be missing in the formulas for Vomma Greek.

4.6. Exercising on Derivation of Vomma Greek

With regard to the identity of formulas for call and put option and with regard to only formal addition or withdrawal of the member $e^{-d\tau}$ in the principal relations of paragraph 4.1., an exercising on the derivation of next possible relations for Vomma Greek is faint.

5. Sense of Gamma Greek, Dual Gamma Greek, Vomma Greek

(quoted according to http://en.wikipedia.org/wiki/Greeks_(finance))

5.1. Practical Use of Gamma Greek

The Gamma Greek measures the rate of change in the Delta Greek with respect to changes in the underlying price. Gamma Greek is the second derivate of the value function with respect to the underlying price. Gamma Greek is important because it corrects for the convexity of value.

When a trader seeks to establish an effective delta-hedge for a portfolio, the trader may also seek to neutralize the portfolio's gamma, as this will ensure that the hedge will be effective over a wider range of underlying price movements.

Of course, in neutralizing the gamma of a portfolio, alpha (the return in excess of the risk-free rate) is reduced.

5.2. Practical Use of Dual Gamma Greek

Although Dual Delta Greek is a primary input into the Black-Scholes model, the overall impact on the value function of an option corresponding to changes in the strike price is generally insignificant and therefore higher-order derivates involving the strike price are not common.

5.3. Practical Use of Vomma Greek

Vomma, **Volga**, **Vega Convexity** or **Vega gamma** measures second order sensitivity to volatility. Vomma Greek is the second derivate of the option value with respect to the volatility, or, stated another way, Vomma Greek measures the rate of change to Vega Greek as volatility changes. With positive Vomma Greek, a position will become long vega as implied volatility increases and short vega as it decreases, which can be scalped in a way analogous to long gamma. And an initially vega-neutral, long-vomma position can be constructed from ratios of options at different strikes. Vomma Greek is positive for options away from the money, and initially increases with distance from the money (but drops off as Vega Greek drops off). (Specifically, Vomma Greek is positive where the usual d_1 and d_2 terms are of the same sign, which is true when $d_2 > 0$ or $d_1 < 0$.)

6. Conclusion

The formulas for Gamma Greek, Dual Gamma Greek, and Vomma Greek were derived through the medium of mathematical data mining tools in statistics education – through the medium of both the differential calculus and the fundamental theorem of integral calculus (see Záškodný,P., Havlíček,I., Budinský,P. (2010-2011)).

Formula for Gamma Greek Γ

$$\Gamma_{CO,PO} = e^{-d\tau} \frac{\rho(d_1)}{S\sigma\sqrt{\tau}}$$

Formula for Dual Gamma Greek Dual Γ

$$\text{Dual } \Gamma_{CO,PO} = e^{-r\tau} \frac{\rho(d_2)}{X\sigma\sqrt{\tau}}$$

Formulas for Vomma Greek Vomma

$$\text{Vomma}_{CO,PO} = Se^{-d\tau}\rho\left(d_1\right)\sqrt{\tau}\,\frac{d_1 d_2}{\sigma} = \nu\,\frac{d_1 d_2}{\sigma}$$

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- http://en.wikipedia.org/wiki/Greeks_(finance)

Application of Mathematical Data Mining Tools – Greeks of Second Order (Vanna, Charm, DvegaDtime)

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Abstract:

An imperative of data mining and a need of cooperation of the human with today´s computers are emphasized by D.A.Keim (Keim, 2002):

"The progress made in hardware technology allows today´s computer systems to store very large amounts of data. Researchers from the University of Berkeley estimate that every year 1 Exabyte (= 1 Million Terabyte) of data are generated, of which a large portion is available in digital form. This means that in the next three years more data will be generated than in all of human history before".

"If the data is presented textually, the amount of data which can be displayed is in range one hundred data items, but this is like a drop in the ocean when dealing with data sets containing millions of data items".

"For data mining to be effective, it is important to include the human in the data exploration process and combine the flexibility, creativity, and general knowledge of the human with the enormous storage capacity and the computational power of today´s computers."

The basic concepts about Data Mining in Statistics Education – see Záškodný,P., Tarábek,P. (2010-2011), ***Data Mining Tools in Statistics Education*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.) The complex data mining tool in statistics education is the Curricular Process of Statistics. The partial data mining tools in statistics education are the Analytical Synthetic Modelling, Statistical and Mathematical Data Mining Tools.

Description of Statistical and Mathematical Data Mining Tools in Statistics Education – see Záškodný,P., Havlíček,I., Budinský,P. (2010-2011), ***Partial Data Mining Tools in Statistics Education – in Greeks and Option Hedging*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.)

Description of Greeks of Combined Second Order in Statistics Education – see Záškodný,P., Havlíček,I., Budinský,P., Hrdlička,L. (2010-2011), ***Where will be used the partial data mining tools in statistics education? In Greeks*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.)

The application of mathematical data mining tools in the light of a deduction of the combined second order greeks "Vanna", "Charm", "DvegaDtime" (based on the Black-Scholes Model) will be described in this paper.

The main principle of paper: Mathematical Data Mining Tools in Statistics Education
The main goal of paper: Deduction of Greeks of Combined Second Order
The procedure of paper: Survey of Greeks of Combined Second Order
Deduction of Vanna Greek
Deduction of Charm Greek
Deduction of DvegaDtime Greek

The results of paper:

1. *Mathematical Derivation of Vanna Greek*
2. *Mathematical Derivation of Charm Greek*
3. *Mathematical Derivation of DvegaDtime Greek*
4. *Practical Use*

Key Words:

Data Mining, Statistics Education, Partial Data Mining Tool – Mathematical Data Mining Tools, Black-Scholes Model, Value Function, Greeks of Combined Second Order, Vanna Greek, Charm Greek, DvegaDtime Greek, Derivation of Formulas for Combined Second Order Greeks "Vanna", "Charm", "DvegaDtime"

1. Survey of Greeks of Combined Second Order

The Black-Scholes model traces the evolution of the option´s key underlying variables in continuous-time. This is done by means of both the standard normal probability densities $\rho(d_1)$, $\rho(d_2)$ and the standard normal distribution functions $N(d_1)$, $N(d_2)$.

The variables d_1, d_2 are connected with Spot price S, Strike price X, Risk-Free Rate r, Annual Dividend Yield d, Time to Maturity τ, and Volatility σ.

Value Function *V* (as Fair Price or as Premium) can be expressed as a function of five quantities $V = \mathrm{f}\,(S, X, r, \tau, \sigma)$

Survey of greeks of the combined second order as survey of the speeds of first order greek change (see Záškodný,P., Havlíček,I., Budinský,P., Hrdlička,L. (2010-2011):

$$\text{Vanna} = \frac{\partial^2 V}{\partial S\,\partial\sigma} = \frac{\partial\Delta}{\partial\sigma} = \frac{\partial\nu}{\partial S} = \text{DdeltaDvol} = \text{DvegaDspot}$$

$$\text{Charm} = \frac{\partial^2 V}{\partial S\,\partial\tau} = \frac{\partial\Delta}{\partial\tau} = \frac{\partial(-\Theta)}{\partial \text{S}} = \text{DdeltaDtime} = \text{D}(-\text{theta})\text{Dspot}$$

$$\text{DvegaDtime} = \frac{\partial^2 V}{\partial\sigma\,\partial\tau} = \frac{\partial(-\Theta)}{\partial\sigma} = \frac{\partial\nu}{\partial\tau} = \text{D}(-\text{theta})\text{Dvol} = \text{DvegaDtime}$$

2. Mathematical Derivation of Vanna Greek (DdeltaDvol)

2.1. Basic Relations

$$\text{Vanna} = \frac{\partial\Delta}{\partial\sigma} = \frac{\partial\nu}{\partial S} = \frac{\partial^2 V}{\partial S\partial\sigma}, V = \langle C\rangle, V = \langle P\rangle$$

The formulas for Vega Greek ν (the call/put option, with/without an annual dividend yield d):

$$\nu = \frac{\partial V}{\partial\sigma} = S\ \rho(d_1)\sqrt{\tau}, \nu = \frac{\partial V}{\partial\sigma} = e^{-d\tau} S\ \rho(d_1)\sqrt{\tau}, V = \langle C\rangle, V = \langle P\rangle$$

$$\nu = \frac{\partial V}{\partial \sigma} = Xe^{-r\tau}\rho(d_2)\sqrt{\tau}, \nu = \frac{\partial V}{\partial \sigma} = e^{d\tau}Xe^{-r\tau}\rho(d_2)\sqrt{\tau}, V = \langle C\rangle, V = \langle P\rangle$$

Next basic relations:

$$\Delta = \frac{\partial V}{\partial S}, V = \langle C\rangle, V = \langle P\rangle$$

$$\langle C\rangle = Se^{-d\tau}N(d_1) - Xe^{-r\tau}N(d_2), \langle P\rangle = Xe^{-r\tau}N(-d_2) - Se^{-d\tau}N(-d_1)$$

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}$$

$$N(d_1) = \int_{-\infty}^{d_1} \rho(d_1)d(d_1), N(d_2) = \int_{-\infty}^{d_2} \rho(d_2)d(d_2)$$

$$N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}, \rho(d_2) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_2^2}{2}}, \rho(d_2) = \rho(d_1)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2}$$

2.2. Implementation of Basic Derivate (with Annual Dividend Yield *d*)

$$\text{Vanna} = \frac{\partial \nu}{\partial S} = \frac{\partial}{\partial S}\left(e^{-d\tau}S\ \rho(d_1)\sqrt{\tau}\right) = e^{-d\tau}\sqrt{\tau}\left(\rho(d_1) + S\frac{\partial \rho(d_1)}{\partial S}\right)$$

2.3. Implementation of Partial Derivates

$$\frac{\partial \rho(d_1)}{\partial S} = \frac{\partial \rho(d_1)}{\partial d_1}\frac{\partial d_1}{\partial S}$$

$$\frac{\partial \rho(d_1)}{\partial d_1} = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}(-d_1) = -d_1\rho(d_1)$$

$$\frac{\partial d_1}{\partial S} = \frac{X/S \; 1/X}{\sigma\sqrt{\tau}} = \frac{1}{S\sigma\sqrt{\tau}}$$

2.4. Adjustment of Implemented Basic Derivate (with Annual Dividend Yield *d*)

$$\text{Vanna} = \frac{\partial \nu}{\partial S} = e^{-d\tau}\sqrt{\tau}\left(\rho(d_1) - Sd_1\rho(d_1)\frac{1}{S\sigma\sqrt{\tau}}\right) = -e^{-d\tau}\rho(d_1)\left(\frac{d_1}{\sigma} - \sqrt{\tau}\right)$$

Using the relation $d_2 = d_1 - \sigma\sqrt{\tau}$ from paragraph 2.1., it is possible to substitute d_1 and to obtain

$$\text{Vanna} = -e^{-d\tau}\rho(d_1)\left(\frac{d_1}{\sigma} - \sqrt{\tau}\right) = -e^{-d\tau}\rho(d_1)\frac{d_2}{\sigma}$$

2.5. Summary

The formulas for Vanna Greek:

$$\text{Vanna} = -e^{-d\tau}\rho(d_1)\frac{d_2}{\sigma} = -\frac{\nu}{S}\frac{d_2}{\sigma\sqrt{\tau}}$$

$$\text{Vanna} = -e^{-d\tau}\rho(d_1)\frac{d_2}{\sigma} = \frac{\nu}{S}\left(1-\frac{d_1}{\sigma\sqrt{\tau}}\right)$$

.

3. Mathematical Derivation of Charm Greek (DdeltaDtime)

3.1. Basic Relations

$$\text{Charm} = \frac{\partial\Delta}{\partial\tau} = -\frac{\partial\Theta}{\partial S} = \frac{\partial^2 V}{\partial S\partial\tau}, V = \langle C\rangle, V = \langle P\rangle$$

The formula for Delta Greek (the call option, with/without an annual dividend yield *d*):

$$\Delta_{CO} = N(d_1), \Delta_{CO} = e^{-d\tau}N(d_1)$$

The formula for Delta Greek (the put option, with/without an annual dividend yield *d*):

$$\Delta_{PO} = -N(-d_1), \Delta_{PO} = -e^{-d\tau}N(-d_1)$$

(see also the relations $N(d_1)+N(-d_1)=1, N(d_2)+N(-d_2)=1$)

Next basic relations:

$$\Theta = -\frac{\partial V}{\partial\tau}, V = \langle C\rangle, V = \langle P\rangle$$

$$\langle C\rangle = Se^{-d\tau}N(d_1) - Xe^{-r\tau}N(d_2), \langle P\rangle = Xe^{-r\tau}N(-d_2) - Se^{-d\tau}N(-d_1)$$

$$d_1 = \frac{\ln S/X + \left(r-d+\sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}$$

$$N(d_1) = \int_{-\infty}^{d_1}\rho(d_1)d(d_1), N(d_2) = \int_{-\infty}^{d_2}\rho(d_2)d(d_2)$$

$$N(d_1)+N(-d_1)=1, N(d_2)+N(-d_2)=1$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}, \rho(d_2) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_2^2}{2}}, \rho(d_2) = \rho(d_1)e^{d_1\sigma\sqrt{\tau}}e^{-\tau\sigma^2/2}$$

3.2. Implementation of Basic Derivate (with Annual Dividend Yield *d*)

$$\mathrm{Charm}_{CO} = \frac{\partial \Delta_{co}}{\partial \tau} = \frac{\partial}{\partial \tau}\left(e^{-d\tau} N(d_1)\right) = -de^{-d\tau} N(d_1) + e^{-d\tau}\frac{\partial N(d_1)}{\partial \tau}$$

$$\mathrm{Charm}_{PO} = \frac{\partial \Delta_{PO}}{\partial \tau} = \frac{\partial}{\partial \tau}\left(-e^{-d\tau}\left(1 - N(d_1)\right)\right) = de^{-d\tau} N(-d_1) + e^{-d\tau}\frac{\partial N(d_1)}{\partial \tau}$$

3.3. Implementation of Partial Derivates

$$\frac{\partial N(d_1)}{\partial \tau} = \frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \tau} = \rho(d_1)\frac{\partial d_1}{\partial \tau}$$

$$\frac{\partial d_1}{\partial \tau} = \frac{-\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{2\sigma\tau\sqrt{\tau}}$$

3.4. Adjustment of Implemented Basic Derivate (with Annual Dividend Yield *d*)

$$\mathrm{Charm}_{CO} = \frac{\partial \Delta_{co}}{\partial \tau} = \frac{\partial}{\partial \tau}\left(e^{-d\tau} N(d_1)\right) = -de^{-d\tau} N(d_1) + e^{-d\tau}\rho(d_1)\frac{\partial d_1}{\partial \tau}$$

$$\mathrm{Charm}_{PO} = \frac{\partial \Delta_{PO}}{\partial \tau} = \frac{\partial}{\partial \tau}\left(-e^{-d\tau}\left(1 - N(d_1)\right)\right) = de^{-d\tau} N(d_1) + e^{-d\tau}\rho(d_1)\frac{\partial d_1}{\partial \tau}$$

$$\frac{\partial d_1}{\partial \tau} = \frac{-\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{2\sigma\tau\sqrt{\tau}} = \frac{2(r-d)\tau - (r-d)\tau - \left(-\sigma^2/2\,\tau\right) - \ln S/X}{2\sigma\tau\sqrt{\tau}}$$

$$d_2 = \frac{\ln S/X + \left(r - d - \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$

$$\frac{\partial d_1}{\partial \tau} = \frac{2(r-d)\tau - d_2\sigma\sqrt{\tau}}{2\sigma\tau\sqrt{\tau}}$$

3.5. Summary

The formulas for Charm Greek:

$$\mathrm{Charm}_{CO} = \frac{\partial \Delta_{co}}{\partial \tau} = -de^{-d\tau} N(d_1) + e^{-d\tau}\rho(d_1)\frac{2(r-d)\tau - d_2\sigma\sqrt{\tau}}{2\sigma\tau\sqrt{\tau}}$$

$$\mathrm{Charm}_{PO} = \frac{\partial \Delta_{PO}}{\partial \tau} = de^{-d\tau} N(-d_1) + e^{-d\tau}\rho(d_1)\frac{2(r-d)\tau - d_2\sigma\sqrt{\tau}}{2\sigma\tau\sqrt{\tau}}$$

4. Mathematical Derivation of DvegaDtime / D(-theta)Dvol

4.1. Basic Relations

$$\text{DvegaDtime} = \frac{\partial \nu}{\partial \tau} = -\frac{\partial \Theta}{\partial \sigma} = \frac{\partial^2 V}{\partial \sigma \partial \tau}, V = \langle C \rangle, V = \langle P \rangle$$

The formulas for Vega Greek ν (the call/put option, with/without an annual dividend yield *d*):

$$\nu = \frac{\partial V}{\partial \sigma} = S\ \rho(d_1)\sqrt{\tau}, \nu = \frac{\partial V}{\partial \sigma} = e^{-d\tau} S\ \rho(d_1)\sqrt{\tau}, V = \langle C \rangle, V = \langle P \rangle$$
$$\nu = \frac{\partial V}{\partial \sigma} = Xe^{-r\tau}\rho(d_2)\sqrt{\tau}, \nu = \frac{\partial V}{\partial \sigma} = e^{d\tau} Xe^{-r\tau}\rho(d_2)\sqrt{\tau}, V = \langle C \rangle, V = \langle P \rangle$$

The formulas for Theta Greek (the call option, with/without an annual dividend yield *d*):

$$\Theta = -\frac{\partial C}{\partial \tau} = -\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}} - rXe^{-r\tau}N(d_2)$$
$$\Theta = -\frac{\partial C}{\partial \tau} = -e^{-d\tau}\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}} - rXe^{-r\tau}N(d_2)$$

The formulas for Theta Greek (the put option, with/without an annual dividend yield *d*):

$$\Theta = -\frac{\partial P}{\partial \tau} = -\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}} - rXe^{-r\tau}N(-d_2)$$
$$\Theta = -\frac{\partial P}{\partial \tau} = -e^{-d\tau}\frac{S\rho(d_1)\sigma}{2\sqrt{\tau}} - rXe^{-r\tau}N(-d_2)$$

(see also relations $N(d_1) + N(-d_1) = 1,\ N(d_2) + N(-d_2) = 1$).

Next basic relations:

$$V = \langle C \rangle, V = \langle P \rangle$$
$$\langle C \rangle = Se^{-d\tau}N(d_1) - Xe^{-r\tau}N(d_2), \langle P \rangle = Xe^{-r\tau}N(-d_2) - Se^{-d\tau}N(-d_1)$$
$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}$$
$$N(d_1) = \int_{-\infty}^{d_1} \rho(d_1)\, d(d_1), N(d_2) = \int_{-\infty}^{d_2} \rho(d_2)\, d(d_2)$$
$$N(d_1) + N(-d_1) = 1, N(d_2) + N(-d_2) = 1$$
$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}, \rho(d_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}}, \rho(d_2) = \rho(d_1) e^{d_1\sigma\sqrt{\tau}} e^{-\tau\sigma^2/2}$$

4.2. Implementation of Basic Derivate (with Annual Dividend Yield *d*)

$$\text{DvegaDtime} = \frac{\partial \nu}{\partial \tau} = \frac{\partial}{\partial \tau}\left(e^{-d\tau} S\ \rho(d_1)\sqrt{\tau}\right) =$$

$$= -de^{-d\tau} S\rho(d_1)\sqrt{\tau} + e^{-d\tau}\frac{1}{2\sqrt{\tau}} S\rho(d_1) + e^{-d\tau} S\sqrt{\tau}\frac{\partial \rho(d_1)}{\partial \tau}$$

4.3. Implementation of Partial Derivates

$$\frac{\partial \rho(d_1)}{\partial \tau} = \frac{\partial \rho(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \tau}$$

$$\frac{\partial \rho(d_1)}{\partial d_1} = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}(-d_1) = -d_1\rho(d_1)$$

$$\frac{\partial d_1}{\partial \tau} = -\frac{1}{2}\frac{\ln S/X}{\sigma\tau\sqrt{\tau}} + \frac{1}{2}\frac{r - d + \sigma^2/2}{\sigma\sqrt{\tau}} = \frac{-\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{2\sigma\tau\sqrt{\tau}}$$

4.4. Adjustment of Implemented Basic Derivate (with Annual Dividend Yield *d*)

An adjustment of the expression $\frac{\partial \rho(d_1)}{\partial \tau} = \frac{\partial \rho(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \tau}$ using the relation $d_2 = d_1 - \sigma\sqrt{\tau}$ from paragraph 4.1.:

$$\frac{\partial \rho(d_1)}{\partial \tau} = -d_1\rho(d_1)\frac{-\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{2\sigma\tau\sqrt{\tau}} = d_1\rho(d_1)\frac{\ln S/X - (r-d)\tau - \sigma^2/2\,\tau}{2\sigma\tau\sqrt{\tau}} =$$

$$= d_1\rho(d_1)\frac{\ln S/X + (r-d)\tau - \sigma^2/2\,\tau - 2(r-d)\tau}{2\sigma\tau\sqrt{\tau}} = d_1\rho(d_1)\left(\frac{d_2}{2\tau} - \frac{2(r-d)}{2\sigma\sqrt{\tau}}\right) =$$

$$= \rho(d_1)\left(\frac{d_1 d_2}{2\tau} - \frac{(r-d)d_1}{\sigma\sqrt{\tau}}\right)$$

An adjustment of the expression for DvegaDtime Greek acquired in paragraph 4.2.:

$$\text{DvegaDtime} = \frac{\partial \nu}{\partial \tau} = -de^{-d\tau} S\rho(d_1)\sqrt{\tau} + e^{-d\tau}\frac{1}{2\sqrt{\tau}} S\rho(d_1) + e^{-d\tau} S\sqrt{\tau}\frac{\partial \rho(d_1)}{\partial \tau} =$$

$$= e^{-d\tau} S\rho(d_1)\sqrt{\tau}\left(-d + \frac{1}{2\tau} + \frac{d_1 d_2}{2\tau} - \frac{(r-d)d_1}{\sigma\sqrt{\tau}}\right)$$

4.5. Summary

$$\text{DvegaDtime} = \frac{\partial \nu}{\partial \tau} = -e^{-d\tau} S \rho(d_1) \sqrt{\tau} \left(d + \frac{(r-d) d_1}{\sigma\sqrt{\tau}} - \frac{1 + d_1 d_2}{2\tau} \right)$$

After substitution Vega Greek by $\nu = e^{-d\tau} S \ \rho(d_1) \sqrt{\tau}$, it is possible to obtain

$$\text{DvegaDtime} = \frac{\partial \nu}{\partial \tau} = -\nu \left(d + \frac{(r-d) d_1}{\sigma\sqrt{\tau}} - \frac{1 + d_1 d_2}{2\tau} \right)$$

5. Sense of Vanna Greek, Charm Greek, DvegaDtime Greek

(quoted according to http://en.wikipedia.org/wiki/Greeks_(finance))

5.1. Practical Use of Vanna Greek

Vanna Greek, also referred to as DvegaDspot and DdeltaDvol, is a second order derivate of the option value, once to the underlying spot price and once to volatility. It is mathematically equivalent to DdeltaDvol, the sensitivity of the option delta with respect to change in volatility; or alternately, the partial of Vega Greek with respect to the underlying instrument's price. Vanna Greek can be a useful sensitivity to monitor when maintaining a delta- or vega-hedged portfolio as Vanna Greek will help the trader to anticipate changes to the effectiveness of a delta-hedge as volatility changes or the effectiveness of a vega-hedge against change in the underlying spot price.

5.2. Practical Use of Charm Greek

Charm Greek or delta decay, measures the instantaneous rate of change of Delta Greek over the passage of time. Charm Greek has also been called DdeltaDtime. Charm Greek can be an important greek to measure/monitor when delta-hedging a position over a weekend. Charm is a second-order derivate of the option value, once to price and once to time. It is also then the (negative) derivate of Theta Greek with respect to the underlying's price.

5.3. Practical Use of DvegaDtime Greek

DvegaDtime Greek, measures the rate of change in the Vega Greek with respect to the passage of time. DvegaDtime is the second derivate of the value function; once to volatility and once to time.

It is common practice to divide the mathematical result of DvegaDtime by 100 times the number of days per year to reduce the value to the percentage change in Vega Greek per one day.

6. Conclusion

The formulas for Vanna Greek, Charm Greek, and DvegaDtime Greek were derived through the medium of mathematical data mining tools in statistics education – through the medium of both the differential calculus and the fundamental theorem of integral calculus (see Záškodný,P., Havlíček,I., Budinský,P. (2010-2011)).

Formulas for Vanna Greek

$$\text{Vanna}_{CO,PO} = -e^{-d\tau}\rho(d_1)\frac{d_2}{\sigma} = -\frac{\nu}{S}\frac{d_2}{\sigma\sqrt{\tau}} = \frac{\nu}{S}\left(1-\frac{d_1}{\sigma\sqrt{\tau}}\right)$$

Formulas for Charm Greek

$$\text{Charm}_{CO} = -de^{-d\tau}N(d_1) + e^{-d\tau}\rho(d_1)\frac{2(r-d)\tau - d_2\sigma\sqrt{\tau}}{2\sigma\tau\sqrt{\tau}}$$

$$\text{Charm}_{PO} = de^{-d\tau}N(-d_1) + e^{-d\tau}\rho(d_1)\frac{2(r-d)\tau - d_2\sigma\sqrt{\tau}}{2\sigma\tau\sqrt{\tau}}$$

Formulas for DvegaDtime Greek

$$\text{DvegaDtime}_{CO,PO} = -e^{-d\tau}S\rho(d_1)\sqrt{\tau}\left(d + \frac{(r-d)d_1}{\sigma\sqrt{\tau}} - \frac{1+d_1d_2}{2\tau}\right)$$

$$\text{DvegaDtime}_{CO,PO} = -\nu\left(d + \frac{(r-d)d_1}{\sigma\sqrt{\tau}} - \frac{1+d_1d_2}{2\tau}\right)$$

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In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk
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- http://en.wikipedia.org/wiki/Greeks_(finance)

Application of Mathematical Data Mining Tools – Greeks of Third Order (Speed, Zomma, Color, Ultima)

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Abstract:

An imperative of data mining and a need of cooperation of the human with today´s computers are emphasized by D.A.Keim (Keim, 2002):

"The progress made in hardware technology allows today´s computer systems to store very large amounts of data. Researchers from the University of Berkeley estimate that every year 1 Exabyte (= 1 Million Terabyte) of data are generated, of which a large portion is available in digital form. This means that in the next three years more data will be generated than in all of human history before".

"If the data is presented textually, the amount of data which can be displayed is in range one hundred data items, but this is like a drop in the ocean when dealing with data sets containing millions of data items".

"For data mining to be effective, it is important to include the human in the data exploration process and combine the flexibility, creativity, and general knowledge of the human with the enormous storage capacity and the computational power of today´s computers."

The basic concepts about Data Mining in Statistics Education – see Záškodný,P., Tarábek,P. (2010-2011), ***Data Mining Tools in Statistics Education*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.) The complex data mining tool in statistics education is the Curricular Process of Statistics. The partial data mining tools in statistics education are the Analytical Synthetic Modelling, Statistical and Mathematical Data Mining Tools.

Description of Statistical and Mathematical Data Mining Tools in Statistics Education – see Záškodný,P., Havlíček,I., Budinský,P. (2010-2011), ***Partial Data Mining Tools in Statistics Education – in Greeks and Option Hedging*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.)

Description of Greeks of Third Order in Statistics Education – see Záškodný,P., Havlíček,I., Budinský,P., Hrdlička,L. (2010-2011), ***Where will be used the partial data mining tools in statistics education? In Greeks*** (In: Tarábek,P., Záškodný,P. (2010-2011), *Educational and Didactic Communication* 2010, Bratislava, Slovak Republic: Didaktis, www.didaktis.sk.)

The application of mathematical data mining tools in the light of a deduction of the third order greeks "Speed", "Zomma", "Color", "Ultima" (based on the Black-Scholes Model) will be described in this paper.

The main principle of paper: Mathematical Data Mining Tools in Statistics Education
The main goal of paper: Deduction of Greeks of Third Order
The procedure of paper: Survey of Greeks of Third Order
Deduction of Speed Greek
Deduction of Zomma Greek
Deduction of Color Greek
Deduction of Ultima Greek

The results of paper:

1. *Mathematical Derivation of Speed Greek*
2. *Mathematical Derivation of Zomma Greek*
3. *Mathematical Derivation of Color Greek*
4. *Mathematical Derivation of Ultima Greek*
5. *Practical Use*

Key Words:

Data Mining, Statistics Education, Partial Data Mining Tool – Mathematical Data Mining Tools, Black-Scholes Model, Value Function, Greeks of Third Order, Speed Greek, Zomma Greek, Color Greek, Ultima Greek, Derivation of Formulas for Third Order Greeks “Speed”, “Zomma”, “Color”, “Ultima”

1. Survey of Greeks of Third Order

The Black-Scholes model traces the evolution of the option´s key underlying variables in continuous-time. This is done by means of both the standard normal probability densities $\rho(d_1)$, $\rho(d_2)$ and the standard normal distribution functions $N(d_1)$, $N(d_2)$.

The variables d_1, d_2 are connected with Spot price S, Strike price X, Risk-Free Rate r, Annual Dividend Yield d, Time to Maturity τ, and Volatility σ.

Value Function *V* (as Fair Price or as Premium) can be expressed as a function of five quantities $V = \mathrm{f}\,(S, X, r, \tau, \sigma)$

Survey of greeks of the third order as survey of the speeds of second order greek change (see Záškodný,P., Havlíček,I., Budinský,P., Hrdlička,L. (2010-2011):

$$\text{Speed} = \frac{\partial^3 V}{\partial S^3} = \frac{\partial \Gamma}{\partial S} = \frac{\partial^2 \Delta}{\partial S^2} = \text{DgammaDspot}$$

$$\text{Zomma} = \frac{\partial^3 V}{\partial S^2 \partial \sigma} = \frac{\partial \Gamma}{\partial \sigma} = \frac{\partial^2 \Delta}{\partial S\, \partial \sigma} = \frac{\partial^2 \nu}{\partial S^2} = \text{DgammaDvol}$$

$$\text{Color} = \frac{\partial^3 V}{\partial S^2 \partial \tau} = \frac{\partial \Gamma}{\partial \tau} = \frac{\partial^2 \Delta}{\partial S\, \partial \tau} = \frac{\partial^2 \left(-\Theta\right)}{\partial S^2} = \text{DgammaDtime}$$

$$\text{Ultima} = \frac{\partial^3 V}{\partial \sigma^3} = \frac{\partial \text{vomma}}{\partial \sigma} = \frac{\partial^2 \nu}{\partial \sigma^2} = \text{DvommaDvol}$$

2. Mathematical Derivation of Speed Greek (DgammaDspot)

2.1. Basic Relations

$$\text{Speed} = \frac{\partial \Gamma}{\partial S} = \frac{\partial^3 V}{\partial S^3}, V = \langle C \rangle, V = \langle P \rangle$$

$$\Gamma_{CO} = \Gamma_{PO} = e^{-d\tau} \frac{\rho(d_1)}{S\sigma\sqrt{\tau}}, \Gamma_{CO} = \Gamma_{PO} = \frac{\rho(d_1)}{S\sigma\sqrt{\tau}}$$

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}$$

2.2. Implementation of Basic Derivate (with Annual Dividend Yield *d*)

$$\text{Speed} = e^{-d\tau}\left(S^{-1} \frac{\partial \rho(d_1)}{\partial S} \frac{1}{\sigma\sqrt{\tau}} + \frac{\rho(d_1)}{\sigma\sqrt{\tau}} \left(-\frac{1}{S^2} \right) \right)$$

2.3. Implementation of Partial Derivates

$$\frac{\partial \rho(d_1)}{\partial S} = \frac{\partial \rho(d_1)}{\partial d_1} \frac{\partial d_1}{\partial S}$$

$$\frac{\partial \rho(d_1)}{\partial d_1} = -d_1 \rho(d_1)$$

$$\frac{\partial d_1}{\partial S} = \frac{\frac{X}{S}\frac{1}{X}}{\sigma\sqrt{\tau}} = \frac{1}{S\sigma\sqrt{\tau}}$$

2.4. Adjustment of Implemented Basic Derivate (with Annual Dividend Yield *d*)

$$\text{Speed} = e^{-d\tau}\left(-\frac{d_1}{S\sigma\sqrt{\tau}} \frac{\rho(d_1)}{S\sigma\sqrt{\tau}} - \frac{1}{S} \frac{\rho(d_1)}{S\sigma\sqrt{\tau}} \right)$$

2.5. Summary

The formula for Speed Greek (the call/put option, with an annual dividend yield):

$$\text{Speed}_{CO,PO} = -e^{-d\tau} \frac{\rho(d_1)}{S^2\sigma\sqrt{\tau}} \left(\frac{d_1}{\sigma\sqrt{\tau}} + 1 \right) = -\frac{\Gamma}{S} \left(\frac{d_1}{\sigma\sqrt{\tau}} + 1 \right)$$

3. Mathematical Derivation of Zomma Greek (DgammaDvol)

3.1. Basic Relations

$$\text{Zomma} = \frac{\partial \Gamma}{\partial \sigma} = \frac{\partial^3 V}{\partial S^2 \partial \sigma}, V = \langle C \rangle, V = \langle P \rangle$$

$$\Gamma_{CO} = \Gamma_{PO} = e^{-d\tau} \frac{\rho(d_1)}{S\sigma\sqrt{\tau}}, \Gamma_{CO} = \Gamma_{PO} = \frac{\rho(d_1)}{S\sigma\sqrt{\tau}}$$

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = \frac{\ln S/X + \left(r - d - \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}$$

3.2. Implementation of Basic Derivate (with Annual Dividend Yield *d*)

$$\text{Zomma} = e^{-d\tau}\left(\frac{\partial \rho(d_1)}{\partial \sigma}\frac{1}{S\sigma\sqrt{\tau}} + \frac{\rho(d_1)}{S\sqrt{\tau}}\left(-\frac{1}{\sigma^2}\right)\right)$$

3.3. Implementation of Partial Derivates

$$\frac{\partial \rho(d_1)}{\partial \sigma} = \frac{\partial \rho(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \sigma}$$

$$\frac{\partial \rho(d_1)}{\partial d_1} = -d_1 \rho(d_1)$$

$$\frac{\partial d_1}{\partial \sigma} = -\frac{\ln S/X}{\sigma^2\sqrt{\tau}} + \frac{\sqrt{\tau}}{2} - \frac{(r-d)\sqrt{\tau}}{\sigma^2}$$

3.4. Adjustment of Implemented Basic Derivate (with Annual Dividend Yield *d*)

$$\frac{\partial \rho(d_1)}{\partial \sigma} = -d_1\rho(d_1)\left(-\frac{1}{\sigma}\right)\left(\frac{\ln S/X}{\sigma\sqrt{\tau}} + \frac{(r-d)\tau}{\sigma\sqrt{\tau}} - \frac{\sigma^2/2\,\tau}{\sigma\sqrt{\tau}}\right) = \frac{\rho(d_1)}{\sigma}d_1 d_2$$

$$\text{Zomma} = e^{-d\tau}\frac{\rho(d_1)}{S\sigma\sqrt{\tau}}\left(\frac{d_1 d_2}{\sigma} - \frac{1}{\sigma}\right)$$

3.5. Summary

The formula for Zomma Greek (the call/put option, with an annual dividend yield):

$$\text{Zomma}_{CO,PO} = \Gamma\left(\frac{d_1 d_2 - 1}{\sigma}\right) = e^{-d\tau}\frac{\rho(d_1)(d_1 d_2 - 1)}{S\sigma^2\sqrt{\tau}}$$

4. Mathematical Derivation of Color Greek (DgammaDtime)

4.1. Basic Relations

$$\text{Color} = \frac{\partial \Gamma}{\partial \tau} = \frac{\partial^3 V}{\partial S^2 \partial \tau}, V = \langle C \rangle, V = \langle P \rangle$$

$$\Gamma_{CO} = \Gamma_{PO} = e^{-d\tau} \frac{\rho(d_1)}{S\sigma\sqrt{\tau}}, \Gamma_{CO} = \Gamma_{PO} = \frac{\rho(d_1)}{S\sigma\sqrt{\tau}}$$

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = \frac{\ln S/X + \left(r - d - \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}$$

4.2. Implementation of Basic Derivate (with Annual Dividend Yield *d*)

$$\text{Color} = e^{-d\tau}(-d)\frac{\rho(d_1)}{S\sigma\sqrt{\tau}} + e^{-d\tau}\frac{\partial \rho(d_1)}{\partial \tau}\frac{1}{S\sigma\sqrt{\tau}} + e^{-d\tau}\rho(d_1)\left(-\frac{1}{2S\sigma\tau\sqrt{\tau}}\right)$$

4.3. Implementation of Partial Derivates

$$\frac{\partial \rho(d_1)}{\partial \tau} = \frac{\partial \rho(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \tau}$$

$$\frac{\partial \rho(d_1)}{\partial d_1} = -d_1\rho(d_1)$$

$$\frac{\partial d_1}{\partial \tau} = \left(\ln S/X + \left(r - d + \sigma^2/2\right)\tau\right)\left(-\frac{1}{2\sigma\tau\sqrt{\tau}}\right) + \frac{1}{\sigma\sqrt{\tau}}\left(r - d + \sigma^2/2\right)$$

4.4. Adjustment of Implemented Basic Derivate (with Annual Dividend Yield *d*)

$$\frac{\partial d_1}{\partial \tau} = \left(\ln S/X + \left(r - d + \sigma^2/2\right)\tau\right)\left(-\frac{1}{2\sigma\tau\sqrt{\tau}}\right) + \frac{1}{\sigma\sqrt{\tau}}\left(r - d + \sigma^2/2\right) =$$

$$= -\frac{d_1}{2\tau} + \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau - \ln S/X}{\sigma\tau\sqrt{\tau}} = -\frac{d_1}{2\tau} + \frac{d_1}{\tau} - \frac{\ln S/X}{\sigma\tau\sqrt{\tau}} =$$

$$= \frac{d_1}{2\tau} - \frac{\ln S/X}{\sigma\tau\sqrt{\tau}}$$

An adjustment of the middle member from implemented basic derivate in paragraph 4.2.:

$$e^{-d\tau}\frac{\partial\rho(d_1)}{\partial\tau}\frac{1}{S\sigma\sqrt{\tau}}=e^{-d\tau}\frac{1}{S\sigma\sqrt{\tau}}\left(-d_1\rho(d_1)\right)\left(\frac{d_1}{2\tau}-\frac{\ln S/X}{\sigma\tau\sqrt{\tau}}\right)=$$

$$=-e^{-d\tau}\frac{\rho(d_1)}{2S\sigma\tau\sqrt{\tau}}2d_1\left(\frac{d_1}{2}-\frac{\ln S/X}{\sigma\sqrt{\tau}}\right)=-e^{-d\tau}\frac{\rho(d_1)}{2S\sigma\tau\sqrt{\tau}}\left(d_1^{\,2}-\frac{2d_1\ln S/X}{\sigma\sqrt{\tau}}\right)$$

A substitution of the middle member from implemented basic derivate in paragraph 4.2 by the adjusted middle member:

$$\text{Color}=-e^{-d\tau}\frac{\rho(d_1)}{2S\sigma\tau\sqrt{\tau}}\left(2d\tau+1+d_1\left(\frac{d_1\sigma\sqrt{\tau}-2\ln S/X}{\sigma\sqrt{\tau}}\right)\right)$$

An adjustment of the expression in last bar by the help of an attestation of equality

$$\frac{d_1\sigma\sqrt{\tau}-2\ln S/X}{\sigma\sqrt{\tau}}=\frac{2(r-d)\tau-d_2\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}$$

$$2(r-d)\tau-d_2\sigma\sqrt{\tau}=2(r-d)\tau-\left(\ln S/X+\left(r-d-\sigma^2/2\right)\tau\right)=$$

$$=2(r-d)\tau-\ln S/X-(r-d)\tau+\sigma^2/2\,\tau=\ln S/X+\left(r-d+\sigma^2/2\right)\tau-2\ln S/X=$$

$$=d_1\sigma\sqrt{\tau}-2\ln S/X$$

By this way, the attestation of equality is implemented.

4.5. Summary

The formulas for Color Greek (the call/put option, with an annual dividend yield):

$$\text{Color}_{CO,PO}=-e^{-d\tau}\frac{\rho(d_1)}{2S\sigma\tau\sqrt{\tau}}\left(2d\tau+1+\frac{2(r-d)\tau-d_2\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}d_1\right)$$

$$\text{Color}_{CO,PO}=-\frac{\Gamma}{2\tau}\left(2d\tau+1+\frac{2(r-d)\tau-d_2\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}d_1\right)$$

5. Mathematical Derivation of Ultima Greek (DvommaDvol)

5.1. Basic Relations

$$\text{Ultima} = \frac{\partial \text{vomma}}{\partial \sigma} = \frac{\partial^3 V}{\partial \sigma^3}, V = \langle C \rangle, V = \langle P \rangle$$

$$\text{Vomma}_{CO,PO} = \frac{\partial \nu}{\partial \sigma} = \frac{\partial^2 V}{\partial \sigma^2} = Se^{-d\tau}\rho(d_1)\sqrt{\tau}\frac{d_1 d_2}{\sigma} = \nu\frac{d_1 d_2}{\sigma}$$

$$\nu_{CO,PO} = \frac{\partial V}{\partial \sigma} = e^{-d\tau} S\ \rho(d_1)\sqrt{\tau}$$

$$d_1 = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = \frac{\ln S/X + \left(r - d - \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$

$$\rho(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}$$

5.2. Implementation of Basic Derivate (with Annual Dividend Yield *d*)

$$\text{Ultima} = Se^{-d\tau}\sqrt{\tau}\left(\frac{\partial \rho(d_1)}{\partial \sigma}\frac{d_1 d_2}{\sigma} + \rho(d_1)\frac{\partial d_1}{\partial \sigma}\frac{d_2}{\sigma} + \rho(d_1)\frac{\partial d_2}{\partial \sigma}\frac{d_1}{\sigma} - \rho(d_1)\frac{d_1 d_2}{\sigma^2}\right) =$$

$$= Se^{-d\tau}\sqrt{\tau}(1+2+3+4)$$

5.3. Implementation of Partial Derivates

$$\frac{\partial \rho(d_1)}{\partial \sigma} = \frac{\partial \rho(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \sigma}$$

$$\frac{\partial \rho(d_1)}{\partial d_1} = -d_1\rho(d_1)$$

$$\frac{\partial d_1}{\partial \sigma} = \frac{\ln S/X + \left(r - d + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}\left(-\frac{1}{\sigma}\right) + \frac{\sigma\tau}{\sigma\sqrt{\tau}} = -\frac{d_1}{\sigma} + \sqrt{\tau}$$

$$\frac{\partial d_2}{\partial \sigma} = \frac{\ln S/X + \left(r - d - \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}\left(-\frac{1}{\sigma}\right) - \frac{\sigma\tau}{\sigma\sqrt{\tau}} = -\frac{d_2}{\sigma} - \sqrt{\tau}$$

5.4. Adjustment of Implemented Basic Derivate (with Annual Dividend Yield *d*)

$$\text{Ultima} = Se^{-d\tau}\sqrt{\tau}\left(\frac{\partial\rho(d_1)}{\partial\sigma}\frac{d_1d_2}{\sigma} + \rho(d_1)\frac{\partial d_1}{\partial\sigma}\frac{d_2}{\sigma} + \rho(d_1)\frac{\partial d_2}{\partial\sigma}\frac{d_1}{\sigma} - \rho(d_1)\frac{d_1d_2}{\sigma^2}\right)$$

$$(1 \quad + \quad 2 \quad + \quad 3 \quad + \quad 4)$$

$$1 = \rho(d_1)\sqrt{\tau}\left(\frac{d_1d_2}{\sigma}\right)^2$$

$$2 = \rho(d_1)\left(-\frac{d_1}{\sigma} + \sqrt{\tau}\right)\frac{d_2}{\sigma}$$

$$3 = \rho(d_1)\left(-\frac{d_2}{\sigma} - \sqrt{\tau}\right)\frac{d_1}{\sigma}$$

$$1+2+3+4 =$$

$$= \rho(d_1)\sqrt{\tau}\left(\frac{d_1d_2}{\sigma}\right)^2 - \rho(d_1)\frac{d_1d_2}{\sigma} + \rho(d_1)\sqrt{\tau}\frac{d_2}{\sigma}$$

$$-\rho(d_1)\frac{d_1d_2}{\sigma} - \rho(d_1)\sqrt{\tau}\frac{d_1}{\sigma} - \rho(d_1)\frac{d_1d_2}{\sigma^2} =$$

$$= \rho(d_1)\left(\frac{d_1d_2}{\sigma}\left(\frac{\sqrt{\tau}d_1d_2}{\sigma} - 2 - \frac{1}{\sigma}\right)\right) + \frac{\sqrt{\tau}}{\sigma}(d_2 - d_1) =$$

$$= \rho(d_1)\frac{d_1d_2\left(\sqrt{\tau}d_1d_2 - 2\sigma - 1\right) + \sigma\sqrt{\tau}(d_2 - d_1)}{\sigma^2}$$

5.5. Summary

The formulas for Ultima Greek (the call/put option, with an annual dividend yield):

$$\text{Ultima}_{CO,PO} = \frac{Se^{-d\tau}\rho(d_1)\sqrt{\tau}}{\sigma^2}\left(d_1d_2\left(\sqrt{\tau}d_1d_2 - 2\sigma - 1\right) + \sigma\sqrt{\tau}(d_2 - d_1)\right)$$

$$\text{Ultima}_{CO,PO} = \frac{\nu}{\sigma^2}\left(d_1d_2\left(\sqrt{\tau}d_1d_2 - 2\sigma - 1\right) + \sigma\sqrt{\tau}(d_2 - d_1)\right)$$

6. Sense of Speed Greek, Zomma Greek, Color Greek, Ultima Greek

(quoted according to http://en.wikipedia.org/wiki/Greeks_(finance))

6.1. Practical Use of Speed Greek

Speed Greek measures the rate of change in Gamma Greek with respect to changes in the underlying price. This is also sometimes referred to as the gamma of the gamma or DgammaDspot. Speed Greek is the third derivate of the value function with respect to the underlying spot price. Speed can be important to monitor when delta-hedging or gamma-hedging a portfolio.

6.2. Practical Use of Zomma Greek

Zomma Greek measures the rate of change of Gamma Greek with respect to changes in volatility. Zomma Greek has also been referred to as DgammaDvol. Zomma Greek is the third derivate of the value function, twice to underlying asset price and once to volatility. Zomma Greek can be a useful sensitivity to monitor when maintaining a gamma-hedged portfolio as Zomma Greek will help the trader to anticipate changes to the effectiveness of the hedge as volatility changes.

6.3. Practical Use of Color Greek

Color Greek, Gamma Decay or DgammaDtime measures the rate of change of gamma over the passage of time. Color Greek is a third-order derivate of the value function, twice to underlying asset price and once to time. Color Greek can be an important sensitivity to monitor when maintaining a gamma-hedged portfolio as it can help the trader to anticipate the effectiveness of the hedge as time passes.

The mathematical result of the formula for Color Greek is expressed in gamma/year. It is often useful to divide this by the number of days per year to arrive at the change in Gamma Greek per day. This use is fairly accurate when the number of days remaining until option expiration is large. When an option nears expiration, Color Greek itself may change quickly, rendering full day estimates of Gamma Greek change inaccurate.

6.4. Practical Use of Ultima Greek

Ultima Greek measures the sensitivity of the option Vomma Greek with respect to change in volatility. Ultima Greek has also been referred to as DvommaDvol. Ultima Greek is a third-order derivate of the value function to volatility.

7. Conclusion

The formulas for Speed Greek, Zomma Greek, Color Greek, and Ultima Greek were derived through the medium of mathematical data mining tools in statistics education – through the medium of both the differential calculus and the fundamental theorem of integral calculus (see Záškodný,P., Havlíček,I., Budinský,P. (2010-2011)).

Formulas for Speed Greek

$$\text{Speed}_{CO,PO} = -e^{-d\tau}\frac{\rho(d_1)}{S^2\sigma\sqrt{\tau}}\left(\frac{d_1}{\sigma\sqrt{\tau}}+1\right) = -\frac{\Gamma}{S}\left(\frac{d_1}{\sigma\sqrt{\tau}}+1\right)$$

Formulas for Zomma Greek

$$\text{Zomma}_{CO,PO} = e^{-d\tau}\frac{\rho(d_1)}{S\sigma^2\sqrt{\tau}}(d_1d_2-1) = \Gamma\left(\frac{d_1d_2-1}{\sigma}\right)$$

Formulas for Color Greek

$$\text{Color}_{CO,PO} = -e^{-d\tau}\frac{\rho(d_1)}{2S\sigma\tau\sqrt{\tau}}\left(2d\tau+1+\frac{2(r-d)\tau-d_2\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}d_1\right)$$

$$\text{Color}_{CO,PO} = -\frac{\Gamma}{2\tau}\left(2d\tau+1+\frac{2(r-d)\tau-d_2\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}d_1\right)$$

Formulas for Ultima Greek

$$\text{Ultima}_{CO,PO} = e^{-d\tau}\frac{S\rho(d_1)\sqrt{\tau}}{\sigma^2}\left(d_1d_2\left(d_1d_2\sqrt{\tau}-2\sigma-1\right)+\sigma\sqrt{\tau}\left(d_2-d_1\right)\right)$$

$$\text{Ultima}_{CO,PO} = \frac{\nu}{\sigma^2}\left(d_1d_2\left(d_1d_2\sqrt{\tau}-2\sigma-1\right)+\sigma\sqrt{\tau}\left(d_2-d_1\right)\right)$$

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- http://en.wikipedia.org/wiki/Greeks_(finance)

Curricular Process in Dosimetry II

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Abstract:
The work presented here shows the importance of the projected curriculum as an integral part of the theory curriculum. Its contents create the typical part of the projected curriculum in the field of dosimetry as an inter-disciplinary subject. Important parts are the accumulation of experimental data, their analysis and evaluation. This contribution attempts to show that not only conceptual and intended curriculums are very important but also the elements of projected curriculum are of importance to show complete features of theory of curriculum.

Keywords:
Didactic communication, Dosimetry, Information, Measurements, Results, Evaluation, Curricular process, Experimental data, Sensitivity

1. Introduction

The contribution presented here is a further continuation of the work „Curricular Process in Dosimetry“ (Singer 2009) published in preceding „Educational and Didactic Communication 2009“, aimed at deepening knowledge in the field of personal dosimetry. The Data Mining way (Záškodný 2009, Záškodný, Novák 2009) and analytical-synthetic ways to the accumulation of data in the analysis and evaluation of characteristics of personal dosemeters were employed. In most cases, this is an internal representation, since the findings are individual in their nature. The difference from many disciplines in the use of experimental data, which are analyzed, and the evaluation of a certain dosemeter is used for the synthesis. In the whole linear process, i.e.:

measurements→results→evaluation→expert report→teaching text→mind of the student→application by the student

the present article deals with the first three steps, which demonstrate the creativity of an expert participant and assistance of further staff members as e.g. employees dealing with irradiation of dosemeters, etc. In further chapters, the three degrees will be shown by way of example of an electronic personal dosemeter, which was examined at the Department of Laboratory Methods and Medical Technology of University of South Bohemia in České Budějovice (Singer 2010).

2. Measurements

The sensitivity and linearity of the DMC 2000 XB dosemeter response was verified in ^{137}Cs gamma-ray beams in accordance with the recommended procedures as described in ISO publications (ISO 1996, ISO 1997). During the verification, the dosemeter was situated on the PMMA phantom, which was a block of tissue-equivalent material 30x30x15 cm^3. The uncertainty was max. 2%. For the assessment of the dose dependence, the dosemeters were irradiated at a distance of 1.5m (perhaps even 0.5, 1, 2m), in dependence over time by dose rates of 1.6 mSv/h to 2.2 Sv/h. A Water phantom was used for comparison with ^{137}Cs.

Energy dependences of the DMC 2000 XB were furthermore measured at the Czech Metrological Institute - Inspectorate for Ionizing Radiation in a similar way, in beams of X-rays (from Seifert 160 kV) of N20 to N300 quality (ISO 1996, ISO 1997), (with uncertainty max. 3%) at a distance of 0.5m and at dose rates of 7.5mSv/h.

3. Results

Figs. 1 and 2 indicate a non-linear dependence within a range of 10 to 50 mSv. The Personal dose equivalent $H_p(10)$ or reference Ambient dose equivalent $H^*(10)$) in percent is not constant but it steeply increases in the case of ^{137}Cs from 5 or 9% to about 20%. This effect is obvious, if the dose rate exhibits changes in this region, according to the nuclear characteristics published by the manufacturer (± 10% or 25%) (MGP 2006). There is a continuous increase between 10 and 100 mSv, which does not exhibit a step response as usual. Above 100 mSv (but also in the region from 10 to 100 mSv in case of a constant dose rate), the deviation is constant within the range of the measurement uncertainty. This fact was verified in a number of repeated measurements. The measurements were also performed in the region from 10 to 100 mSv on a water phantom. The response for gamma photons from ^{137}Cs does not differ from the measurement on a PMMA phantom by more than the standard deviation calculated for measurements on a given phantom (Fig. 1).

In this region of doses (about monthly investigation level of $H_p(10)$ = 0.5 mSv) the energy dependence (see Table), i.e. difference between the reference and dose equivalent measured in an energy interval of 16 to 1332 keV, ranges between minus 10.9% and plus 37.5% and for doses exceeding 20 mSv up to about 51%. This “asymmetric” energy dependence (see Table) indicates a possibility of the dosemeter use within the whole energy range after a mathematical correction provided in such a way that the energy dependence is “symmetrical” and smaller than ± 25%. The correction in this dose region will be of k = 0.88, which will present the actual dose between 0.78 and 1.22 (including the standard error of about ±1%) when using the formula

$$D_{actual} = k * D_{measured}$$

For higher doses, where the energy dependence exerts higher values, “k” should be more reduced, for example to a value of 0.83, to avoid the possibility that the deviation exceeds ± 25% of the dose equivalent calculated value from the actual value.

The directional dependence is also not symmetrical, which is obvious in terms of the asymmetry of detectors and correction filters in the dosemeter. In this case, it is not suitable to introduce the correction, since the directional dependence is likely to be different for different energies. For softer photons, there will be a more considerable dependence and thus, in practice, the most suitable approach is including the directional dependence into the measurement error. Due to the fact that in practice, the dosemeter is not irradiated in one direction for the whole irradiation period, the error will be considerably smaller compared with that resulting from Fig. 3.

Table Energy dependence
Reference dose $R = H_p(10)$ within range of 0.21 to 1 mSv
(9 to 14 dosemeters on PMMA for each Energy) (Standard deviation ± 0.8 to 1.4%)

E[keV]	Measured (*D*) / Reference(*R*)
16	0.897
25	1.180
33	1.375
48	1.074
65	1.083
83	1.018
100	0.917
118	0.891
164	0.897
207	0.980
250	0.990
^{137}Cs	1.091
^{60}Co	0.948

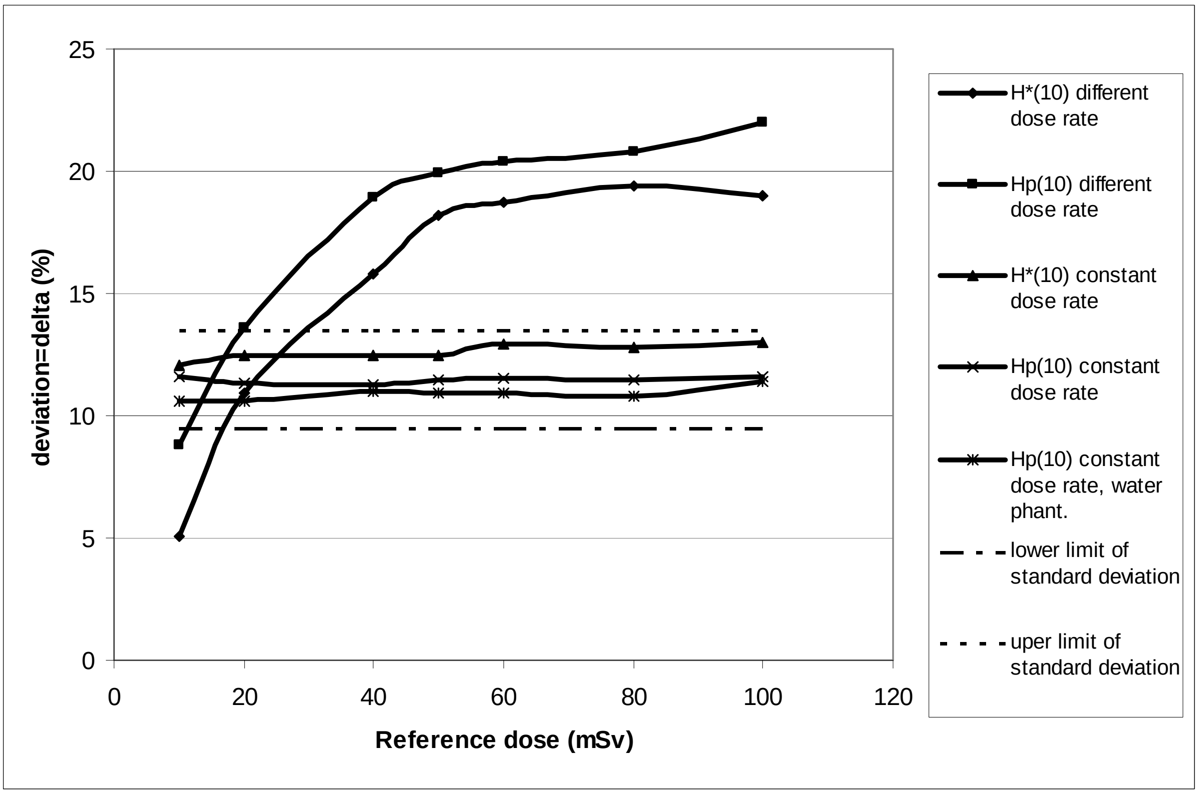

Fig. 1 Dose dependence for ^{137}Cs gamma ray-energy within range where it is nonlinear
(4 to 12 dosemeters for each Dose and each curve)
(*R* – reference dose, δ – deviation of measured dose) (standard deviation ±0.3 to 2.7 %)

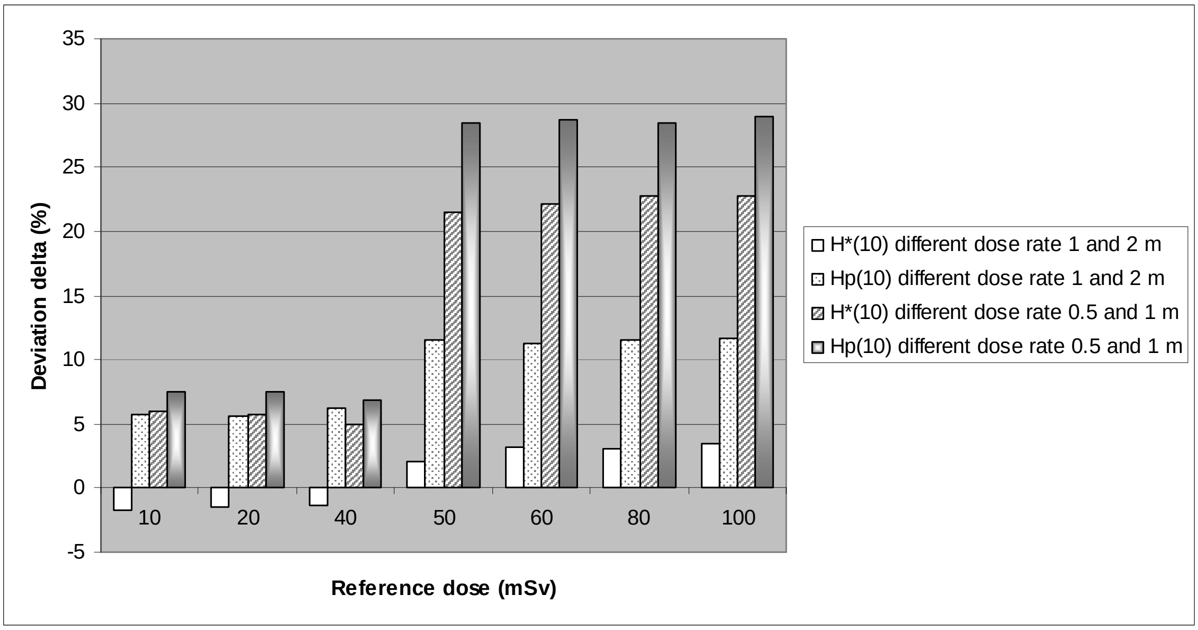

Fig 2 Dose dependence for ^{137}Cs gamma-ray energy within non-linear range
(4 dosemeters for each Dose and each dependence)
(*R* – reference dose, δ – deviation of measured dose, from table 4 f))
(standard deviation ±0.3 to 2.7 %)

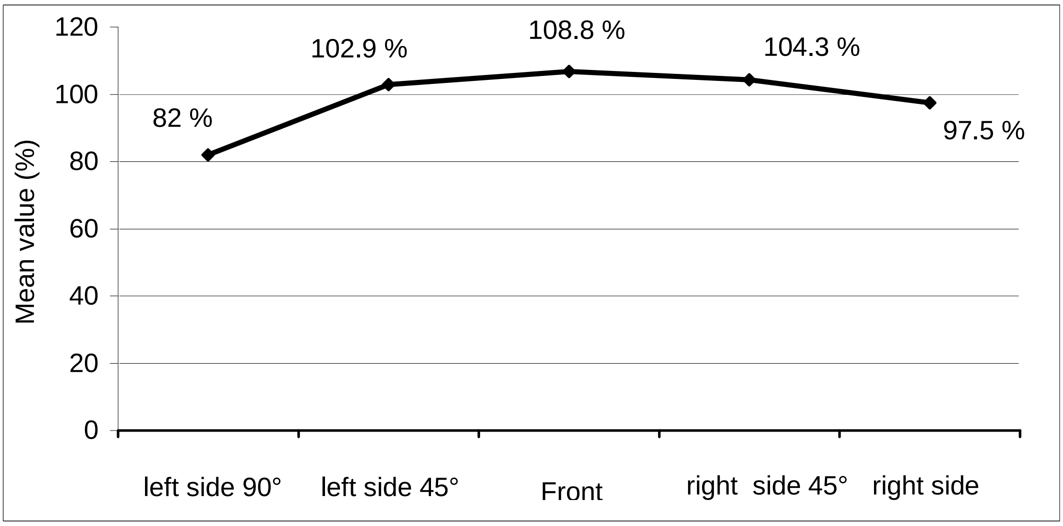

Fig. 3 Directional dependence (from left side 90^{o} to right side 90^{o}) for ^{137}Cs within range of doses 0.1 to 0.5 mSv, (dose rate 1.6 mSv/h) i.e. between monthly recording and investigation levels. Value 100% (in front direction) is $H_p(10)$ - reference

4. Evaluation

The aim of this work is the assessment of the behaviour of a personal dosemeter in applications where the dose rate varies, as for example in a nuclear plant or in a hospital. The PMMA phantom was used like a close tissue-equivalent material, despite the difference of some parts of its energy dependence from e.g. the use of a water phantom.

Results of measuring personal doses by any dosemeter present problems, which are not simple. Any dosemeter, and thus also the electronic personal dosemeter, has its special features, particularly if it consists of multiple components [6,7]. The dose dependence is not linear and energy and directional dependences are not symmetrical. This nonlinearity does not exceed the limits given by tubular curves (IAEA 2007) and after a correction on energy dependence, the dosemeter can be effectively used in practice. It is not possible to read more accurately the value of the personal dose equivalent direct from a dosemeter. For a higher accuracy it is necessary to use LDM 220 reader and to access the dose history and the dose rate history on a computer. In practice, this is possible at higher doses (for example over limit) only, that are for reasons of the radiation protection subject to further analysis.

5. Conclusion

The purpose of this contribution was to consider a method of "the curricular process of an interdisciplinary branch" the ionizing radiation dosimetry being undoubtedly such a type of branch. Former authors studied the didactic communication in physics and its curricular process, which is rather complex in an interdisciplinary branch including mathematics, physics, chemistry, biology (medicine), electronics and use of transformations T1 to T5. Some of these transformations can be actually missing, particularly if technical tools (as e.g. in electronics, etc.) are used instead of a scientific system. Thus, as I believe, it is suitable to use a linear order model (see Singer 2009) with access information and branch outputs.

The work deeply analyzes individual parts of the linear order model (Singer 2009) by way of a specific example taken from research dosimetric practice. Experimental data were analyzed and evaluated in first three steps of the model mentioned.

Measurement 1→ → ┐
2→ → ┐
3→ → Result: Table → ┐
4→ → Result: Fig. 1 → Evaluation
.→ → Result: Fig. 2 → ┘
.→ → Result: Fig. 3 → ┘
.→ → ┘
n→→ ┘

It is obvious that in the example used (but also in general) the linear system is narrowed from the first step to the third one i.e. it continues from a large number of measurements (1 to n) to the final uniform evaluation.

The research of further steps, i.e. "compilation of teaching texts" to "the use by the student" will be implemented in further parts of the Curricular Process in Dosimetry.

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Research of Personality Aspects of Prosocial Behaviour

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Abstract:

Urgency of the proposed project consists in an effort of a detailed theoretical and empirical examination of the prosocial behaviour problems, which even though considered as one of the noblest displays of human personality as such, has been quite undeservedly neglected in the tradition of the Czech research up to the present. The main objective of the project lies in the theoretical and empirical research of the personality aspects of prosocial behaviour problems in adolescents and its relevant correlators (altruism, empathy, value orientations, social and personal norms) in the context of the gender problems, voluntarism, and the family system perception. Especial emphasis is put in the project on possibilities of effective development of deficient prosocial behaviour in adolescents.

In the light of curricular process (see P.Tarabek, P.Zaskodny (2009-2010), Educational & Didactic Communication 2009, ISBN 978-80-89160-69-3) the presented project can be taken as a part of conceptual curriculum (communicable scientific system of relevant scientific branch).

Keywords:

Prosocial behaviour, human personality, relevant correlators, altruism, empathy, value orientations, social norms, personal norms,

1. Outline of the present state of the analysed problem

Urgency of the proposed project consists in an effort of a detailed theoretical and empirical examination of the prosocial behaviour problems, which even though considered as one of the noblest displays of human personality as such, has been quite undeservedly neglected in the tradition of the Czech research up to the present. The main objective of the project lies in the theoretical and empirical research of the personality aspects of prosocial behaviour problems in adolescents and its relevant correlators (altruism, empathy, value orientations, social and personal norms) in the context of the gender problems, voluntarism, and the family system perception. Especial emphasis is put in the project on possibilities of effective development of deficient prosocial behaviour in adolescents.

Within the scope of the conceptual analysis it is advisable to distinguish prosocial behaviour from other related terms, which are **helping and altruism**. H. Bierhoff (2006) considers the concept of helping the most general one, which is superordinate to both the terms and which covers all forms of mutual human support. He defines prosocial behaviour as being aimed at improving the situation of another person, while the help provider is not obliged to render help on the grounds of his/her profession and the help receiver is neither an organization nor an institution, but it is an individual. H. Bierhoff perceives altruism as an independent form of prosocial behaviour motivated by the benefit for another person and inner rewards for the help provider.

Up to the present, no unified theory of prosocial behaviour has been formulated yet. Prosocial behaviour is currently explained by means of approaches based on the problems of norms and values internalization, costs and profits analysis, empathic-altruistic motivation and the process of problem solving.

2. The conceptions of prosocial behaviour

1) In the conception of prosocial behaviour as a **result of social norms internalization**, the process of helping is influenced by cultural norms. One of the significant norms stimulating prosocial behaviour is the norm of social reciprocity, i.e. the principle help those who already helped us themselves, and not to harm the one who helped us before (Staub, 1978). This norm operates both on individual as well as group level and it is particularly implemented among family members, friends, colleagues or community members (for more details see e.g. Van Vugt, Van Lange et al., 2006). A broader framework for the understanding of prosocial behaviour is offered by the exchange theory (Výrost, Slaměník et al., 2008), nowadays already regarded as a classic theory on the basis of which other modifications and constructs originated later, such as the belief in a just world or the norm of justice and equality (see e.g. Lerner, 1980; Walter, Walter, Berscheid, 1978). Prosocial behaviour is often motivated by the norm of social responsibility, which orders to care for people who are not able to manage their own (for more details see e.g. Hewstone, Stroebe, 2006). According to S. H. Schwartze (1977), prosocial behaviour can be motivated by the personality norms, i.e. convictions and values that in the course of socialization had created internalized inner behavioural standards. S. H. Schwartz and J.A. Howardovou (1982) incorporated all these prerequisites into a multi-step model of a decision making process in an emergency situation.

2) In the conception of the prosocial behaviour as a result of costs and profits analysis it is presumed that people practically always weigh personal costs and profits related to the implementation of certain activity carried out with regard to satisfying the needs of others. If the costs are low, the need for a reward is not particularly big. The costs for helping are usually associated with the person providing the help, his/her time, effort, money, potential threats etc. As the costs increase, the willingness of this person to help declines. The intrapersonal conflict of the potentially helping person is characterized by weighing expected benefits (for the one in need and for the one providing help himself) and expected costs. In their initial reflections and the model of helping, I. M. Piliavin and J. Piliavin (1972) included not only the profit and costs of helping, but also the profit and costs of declining help. According to this model, a help is expected if the benefits from helping are large or if the costs of declining help are large.

3) Prosocial behaviour is explained also as a **result of the empathic-altruistic motive**. C. D. Batson (1991) based his hypothesis of altruistic help motivated only by the benefit of the person in need, not by self-benefits, by an emotionally-cognitive response of an individual observing another person in a situation of adversity, i.e. on empathy. He perceives empathy as a care in terms of taking the one's in need emotional and cognitive perspective. By means of the key elements, C. D. Batson constituted three different ways leading the observer from the moment of noticing another person's need to the behavioural reply, i.e. diversely effective help. It concerns two ways motivated by egoistic incentives, i.e. reward-seeking, and the reduction of excitement, and the way stimulated by altruistic motives on the basis of invoked empathy – empathically evoked.

4) In contemporary psychology, prosocial behaviour is also understood as a **result of the decision-making process**. The models of intervention by J. M. Darley and B. Latané (1968); J. A. Piliavin, J. F. Dovidio, S. L. Gaertner and R. D. Clark (1982), and the normative model of intervention by S. H. Schwartz and J. Howard (1982) are the best-known.

Two mutually independent socially cognitive processes are generally recognized as crucial for the development of prosocial behaviour. It concerns **moral reasoning** (for more details see Kohlberg, 1976; Colby, Kohlberg, 1987), and **empathy** (for more details see Hoffman, 1976, 1977, 1987, 2000). Broadly accepted model of socialization of prosocial behaviour is Cialdini's model (Cialdini, Kenrick, 1976; Cialdini, Kenrick, Baumann, 1982). R.B. Cialdini and his collaborators claims (Cialdini et al. 1982) claim that in the course of maturing children experience three stages related to helping. Originally external norms, which used to motivate helping, have been internalized, motivation to help has been shifted from external to internal space.

One of the prosocial behaviour forms is **altruism**. The majority of present-day psychologists employing the term altruism take the opinion of universal egoism, i.e. that the entire human behaviour, including the help we offer others, is a product serving egoistic motives. According to C. D. Batson, it is possible to distinguish three types of pseudo-altruistic approaches, i.e. 1) the conception of altruism as a helping behaviour disregarding motivation (see e.g. Dawkins, 1976; Hamilton, 1964, 1971; Trivers, 1971; Wilson, 1975; Rushton, 1980), 2) the conception of altruism as an act of helping to obtain an internal rather than external reward (see e.g. Bar-Tal et al., 1982; Schwartz, 1977; Schwartz, Howard, 1982; Cialdini, Baumann, Kenrick, 1981; Cialdini, Darby, Vincent, 1973; Cialdini, Kenrick, 1976), and 3) the conception of altruism as an act of helping motivated by the effort to reduce negative inner state evoked by observing afflictions of others (see e.g. Piliavin, Piliavin, 1973; Piliavin, Dovidio, Geartnerem, Clarck, 1981). D. Batson (1991) believes that none of these three viewpoints describes what is traditionally meant by altruism because every example is primarily focused on one's own benefits of some form.

Prosocial behaviour is closely related to the problems of **empathy**. In the course of its history, the term empathy has undergone a long and complicated conceptual development and even though having a crucial psychological relevance, still it represents rather contradicting and inconsistently approached construct (see Vispé, 1987; Verducci, 2000). C. Duan and C. E. Hill (1996) point out certain problems in the conception of empathy to be caused by the fact that in the psychological literature, with regard to the generally used basic classification of the psychological phenomena, empathy is understood as 1) personality trait (see e.g. Davis, 1983, 1996 and others), 2) situation-specific psychological state (see e.g. Rogers 1957) or as 3) multistage experiential process (see e.g. Barret-Lennard, 1981).

In contemporary psychology, it is commonly distinguished between emotional (affective) empathy and cognitive empathy although still relatively little perspicuous knowledge about the relation between both the types of empathy exists. In contemporary psychology, a dimension-like nature of emotional as well as cognitive empathy, proved also through empirical observation (see e.g. Brehmsová, 1989), is presumed. Some authors proved the emotional empathy state to be a mediator of the helping behaviour (see e.g. Eisenberg, Miller, 1987), while the cognitive empathic state alters the process of attribution of other‘s behaviour (see e.g. Gould, Sigall, 1977). Other authors believe that cognitive empathy is a prerequisite for the development of emotional empathy (see. e.g. Staub, 1987), yet others assume complicated interactive relations among them (see e.g. Hoffman, 1987).

With regard to the understanding of the basic psychological quality of empathy, it is possible to differentiate between several empathy conceptions which primarily emphasize its emotional nature; conceptions which perceive empathy predominantly as a cognitive construct, conceptions stressing the emotionally cognitive trait of this expression, conceptions of empathy which principally emphasize its multistage character and 5) conceptions of empathy stressing its multidimensional structure.

3. The conceptions of empathy

1) **Emotional conceptions of empathy** primarily emphasize its affective aspects, and thus historically draw on theorists having published their treatises on the problems of sympathy. In these conceptions, empathy is in essence understood as an emotional reaction of an observer to another person's observed emotions (see e.g. Eisenberg, Strayer, 1987). In these conceptions, the mechanism of getting into the spirit of others is particularly emphasized.

2) **Cognitive conceptions of empathy** define this phenomenon by means of cognitive aspects and at the same time emphasize the mechanism of projecting oneself into the mind of others, i.e. taking the role which is based on symbolic or imagined projection of an individual to another's position, taking his/her roles and attitudes. In these conceptions, empathy is in essence conceived as a perceptive quality, the ability of social insight, or even as a communicative process. It is quite often that empathy in these conceptions is described as a synonym for the concept of intellectual role-taking, or the concept of an observer perspective-taking (see e.g. Carkhuff, Truax, 1969; Hogan, 1969).

3) In **emotionally cognitive conceptions,** predominantly a narrow interconnection of emotional and cognitive processes is emphasized, where cognitive processes play a mediating role in the rise of emotional reactions. In the course of an emphatic process, an individual observes another person and based on the cognitive interpretation of meanings, an emotional reaction, resulting in his perspective taking, arises (see e.g. Rogers, 1975; Strayer, 1987).

4) **Multistage conceptions perceive empathy** as a process concurrently encompassing cognitive as well as emotional aspects and which is possible to divide into mutually connected, yet

clearly distinguished phases or stages. These conceptions of empathy were developed in the sphere of psychotherapy, in which empathy generally represents a key term. The biggest attention was devoted to G. T. Barrett-Lennarda's cyclical model of empathy (1981).

5) **Multidimensional conceptions of empathy** define empathy as a phenomenon which apart from affective and cognitive items encompasses also other dimensions or components. It is particularly M. H. Davis' conceptions (1980, 1983, 1996) which can be ranked among the multidimensional conceptions of empathy. In contemporary psychology, the above Davis' conceptions are considered the most elaborate. According to this author, empathy consists of four mutually related, yet separate basic components or tendencies, i.e. 1) perspective taking, which reflects the tendency to take others' viewpoint which is based on non-egocentric thinking and a tendency to adjust the psychological opinion of others in everyday life, 2) empathic concern, which incorporates the feelings of sympathy, warmth, compassion, concern about adversities of others that can be revealed in altruistic behaviour, **fantasy**, which concerns the tendency to employ imaginative-like ways to transform oneself into the feelings and behaviour of imaginary fiction, play, or movie characters, and 4) personal distress, which reflects the level of self-oriented feelings of anxiety and unease in intensive interpersonal situations and to experience others' feelings of discomfort.

In an effort to arrange semantically varied, or on the contrary, semantically overlapping constructs which in a rather simplified fashion in the specialized literature used to be termed as empathy, and which had resulted in a numerous theoretical ambiguities as well as contradictory research results, M. H. Davis (1996), one of the most outstanding present-day theorists in this field, created what is called an **organizational model of empathy.**

According to M. H. Davis' **organizational model**, it is possible to distinguish between emotional and cognitive outcomes of empathy. Cognitive outcomes can be differentiated into parallel and reactive outcomes. A parallel emotional outcome can only be considered under the condition that the observer's emotional experience corresponds to or at least is drawn nearer to the feelings of the one observed. Contrary to the parallel outcome, the reactive outcome is a state in which the observer's emotional experience is not a mere reflection or a rough reproduction of feelings of the one observed, but it also includes further observer's emotional responses to the reaction of the observed person. With relation to the reactive outcome, a significant attention is drawn particularly to the construct - due to its nature ranking among reactive responses – which is the feeling of compassion with others, in literature either termed as sympathy, empathy, or empathic concern. At the same time, it is possible to observe a growth of interest in another construct, which can be regarded as self-oriented reactive outcome, i.e. personal distress. This construct is characterized by anxiety or unease, felt for the observed people in situation of considerable strain, emergency or distress. According to M. H. Davis' organizational model, affective outcomes of empathic processes result from the interaction of personal and situational factors, but also from empathic processes which differ in the level of cognitive influences. For instance, M. L. Hoffman (1984a, 1984b) distinguishes 6 methods of shaping the observer's reaction. These methods differ in the level of the cognitive development. It concerns 1) motor commands of facial expressions, 2) primary cyclical reaction, 3) classical conditioning, 4) direct association, 5) language-mediated association, and 6) role taking. Specific affective outcomes are dependent on the level of one's cognitive development, predominantly on the level of his cognitive appreciation of others. Cognitive outcomes of emphatic processes may be defined as certain forms of judgments, opinions, evaluation and convictions related to others. Earlier research had often perceived empathy as an accuracy of perception of others, or as a non-verbal sensitivity; later on, empathy was also examined e.g. in relation to attributive judgments.

4. The prosocial personality

The research project is closely related to the search for **prosocial personality traits** or what is called the **prosocial personality**. According to L. A. Pennera et al (2005) the research into prosocial personality traits in essence proceeds in three different directions. It concerns the research of 1) the personality traits, 2) the style of relations, 3) motives and values. Prosocial personality traits represent the permanent aspects of one's nature, which are revealed in one's opinions, feelings and behaviour. Although one's behaviour is also influenced by the situational context, a considerable degree of consistency in people's thoughts, feelings and behaviour is caused by the personality traits. Situational variables, as described in numerous models of prosocial behaviour, constitute a very important cause of social behaviour. As soon as one is in a particular situation or with certain people, his/her behaviour will be influenced by the characteristics of the given situation and by the behaviour of others in the given situation involved; nevertheless, traits and other personality variables also play their role (see Rushton, 1984, Penner, 2004; Snyder, 1992). In connection with the five-factor model (Costa, McCrae, 1994; Goldberg, 1993; Ashton, Paunonen, Helmes, Douglas, 1998), the traits of amiability and conscientiousness are directly related to the prosocial personality. Amiability is connected with trust, reliability and idealism. A higher degree of amiability is typical of those with a greater extent of cooperation-like concentration on others, those, who provide voluntary help to others more often (Carlo, Okun, Knight, De Guzman, 2005). A higher degree of conscientiousness is positively connected with competence and reliability. Another approach to the search of prosocial personality traits consists in the identification of a set of those personality attributes that mutually correlate on a high level, and all are related to prosocial thoughts, feelings, and behaviour. A prosocial personality is possible to describe e.g. by means of higher level of empathy, willingness to accept responsibility for his/her behaviour and for the welfare of others, bigger responsibility and capability to worry about others and to feel attached to them, higher level of self-control and self-efficacy.

Prosocial personality attributes are closely related to the **value orientation**. According to S. H. Schwartz (1992), values are ideas of what is for one desirable. Values govern the codes of conduct, appraisal of others, and explanation of events. The basic content aspect differentiating the values is the type of motivation objectives expressing the values. The typology of different contents of individual values includes, in the form of conscious objectives, three universal conditions of human existence: 1) the needs of individuals as biological organisms; 2) the requirements for co-ordinated social interaction; and 3) the needs for survival and a group benefit. With reference to S. H. Schwartz's classification of values, it is particularly the universality (justice) and benevolence (kindness) which cohere with prosocial behaviour.

5. Next prosocial behaviour attributes

It can be presumed that the problems of prosocial behaviour are significantly related to the quality of the **elemental family environment perception** of adolescents, which in the Czech research has not been systematically studied yet. It is possible to suppose the tendency of adolescents' deficient prosocial behaviour and its significant correlators to be strongly influenced by less favourable cognitive-emotional situation of the elemental family environment. Valid psychometric assessment of the level and quality of the family environment appears to be rather complicated theoretical as well as methodological problem concerning particularly the definition of the principal quantifiable dimensions of family systems (see e.g. Nevolová 1991; Kováliková 1996). In order to research and implement a diagnostic screening of a family environment it is appropriate to create and use also suitable multidimensional questionnaire-employed method capturing particularly socially perceptive aspects of its members. For this purpose, it is necessary to operationalize the knowledge resulting from a numerous inspiring models of foreign as well as domestic provenance, such as the models of family functioning by W. R. Beavers, D. H. Olson, J. Riskin , R. H. Moos, D. Kantor and W. Lehr, S. Minuchina, McMaster model, local, e.g. I. Plaňava's model of coexistence, or family model by J. Dunovský (see Dunovský, 1986; Plaňava, 2000; Simon, Stierlin, 1995; Mlčák, 2000; Sobotková, 2001). In order to diagnose family environment perceptions, it is possible to employ R. H. Moos' family model ((Moos, Moos 1981). This model was implemented into items of the standardized Family Environment Scale (FES – R) created by R. H. Moos, P. M. Insel and B. Mumprey (1974, it is also at disposal in its Slovak modification as Škála rodinného prostredia (ŠPR) [The scale of family environment] by M. Hargašová a T. Kollárik (1980).

Prosocial behaviour is substantially influenced by the problems of **gender**. At present, this term is used to define social and cultural differences in the behaviour and experiences of men and women. According to R. K. Unger (2001) it is a reflection of social understanding of the phenomena of masculinity and femininity, which on individual level can be understood as an internalized complex of characteristic features constituting the gender identity of a man and woman personality. On this level, gender can also be expressed as a system of gender convictions. Gender convictions are related to a group membership, i.e. men and women's identification with their own group members of the same sex, or with their formal membership in broader social groups. Gender convictions are closely related to the different status of men and women in society, including the remaining superiority of one sex over the other, but also to another characteristic features of men and women which include the age, membership in ethnical or racial groups, certain social class integration, religious and political orientation or geographic aspects (for more details see e.g. Bem, 1993; Wyrobková, 2005; Janošová, 2007).

According to A. H. Eagly and M. Crowly (1986) female gender role of women includes norms which activate certain forms of helping. Women are expected to care for personal and emotional needs of others. Such care is expected to be void of the personal service routine and facilitate others' goals reaching. The requirement to serve others is particularly strong inside a family and predominantly concerns intimate relations, such as friendship. The research proves women, due to their helpfulness, kind-heartedness, compassion and willingness to sacrifice themselves, to have been appraised more favourably than men. These qualities seemed more desirable to women rather than men. In woman's friendships more expressions of kindness, emotional and information support was noticed than in cases of man's friendships. On the contrary, male gender role, especially its traditional forms, results in different forms of helping. One of the forms is heroic behaviour, i.e. altruistic behaviour protecting others from damage, performed at the expense of one's own risk. Heroic behaviour is much more appreciated by men than women; generally, male-heroes rather than female-heroine are considered. The research into gender stereotypes provides only limited support for relating the masculinity and heroism. Men rather than women are ascribed such qualities as the willingness to risk, adventurous character, cool self-control in predicaments, ability to be pressure-resistant, and often these qualities are mentioned as men-required. These attributes predispose men to act as heroes, particularly in extreme, unusual conditions. The heroic form of helping presumes men to be willing to help in different situations than woman.

Prosocial tendencies are strongly implemented in **voluntarism** which above all constitutes one of the cornerstones of a society, helps to maintain and strengthen such human values as sociability, concern about others, and service rendered to others. The most common form of voluntarism is carried out by people who, free of charge, devote their time and effort to a charity, religious or other service providing organization (Wilson, 2000). A. Omoto and M. Snyder (1995) specify the differences between spontaneous and planned long-term help pointing out the short-term help to be provided optionally. The majority of cases include certain implicit or explicit feeling on the side of potential help providers that a help should be offered to the potential beneficiary as a result of a normative expectation. These feelings of an obligation can be triggered by verbal or non-verbal behaviour of an unknown person in adversity, personal relationship between the help provider and the beneficiary, or friendly and family relations (see e. g Dovidio et al., 2006).

By means of thein research, L. A. Penner and M. A. Finkelstein (1998) and others try to find some explanation of circumstances initiating the inception of voluntary activities and their perseverance despite substantial costs and effort invested in the longer-time period. Reasons are usually sought in the personality and demographic features in interpersonal relations and situational factors. As regards the demographic features, three variables are often examined: a) education, b) income, c) sex (see e.g. Penner and others., 2005; Wilson, 2000). It is, however, the personality traits and motivation which play decisive role in the decision-making. Many volunteers consider their activities significant also from the viewpoint of self-fulfilment, self-confidence boosting, and the feeling of being useful to others. Voluntarism develops new skills, abilities needed to provide a help. Having new experience, contacts, but also friends is self-rewarding. Last but not least, voluntary activities are a meaningful way of spending one's free time. An important role in the decision-making process of whether or not to become a volunteer plays certain set of personality dispositions, such as also empathy. L. Penner and his colleagues (Penner, 2002; Penner, Fritzsche, Craiger, Freifeld, 1995; Penner, Midili, Kegelmeyer, 1997) for instance proved volunteers to possess greater amount of the prosocial personality traits, i.e. empathy, altruism, moral reasoning. Two significant theoretical models attempted to identify factors supporting voluntary work resumption after a longer time-period: Omoto and Snyder (1995, 2002) developed the Volunteer Process Model, J. A. Piliavian and her colleagues (Piliavin, Grube, 2000; Piliavin, Callero and Grube, 2002) created the Role Identity Model.

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Conclusion of Monograph "Eduactional & Didactic Communication 2010": Survey of Data Mining Tools in Education

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1. Complex Data Mining Tool

Complex Data Mining Tool – Curricular Process

Structure of Curricular Process:

Conceptual Curriculum (Communicable Scientific System of Relevant Scientific Branch)
Intended Curriculum (Educational System)
Projected Curriculum (Instructional System, e.g. Textbook)
Implemented Curriculum 1 (Preparedness of Educator for Instruction, Education)
Implemented Curriculum 2 (Results of Instruction, Education)
Attained Curriculum (Applicable Results of Instruction, Education)

2. Partial Data Mining Tools

Partial Data Mining Tool 1 – Analytical Synthetic Modeling
Partial Data Mining Tool 2 – Triangular Modeling of Concepts
Partial Data Mining Tool 3 – Matrix Modeling
Partial Data Mining Tool 4 – Statistical Data Mining Tools
Partial Data mining Tool 5 – Mathematical Data Mining Tools

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APPENDIX: Announcement about the cooperation with Mr. Mohamed Salem Soudani

Authors: Petr Prochazka
SEI Co. (Safety Engineering & Improvement Company), Czech Republic

The Curriculum Studies Research Group has accepted the cooperation with Mr. **Mohamed Salem Soudani**, an education expert in English teaching in Morocco.

Mr. Soudani participated and leaded various educational and research projects in Morocco and abroad. He attended many English teacher positions in many schools in Morocco and is also experienced in professional education in private companies.

Mr. Soudani will enrich our research group with the research topic “Evaluation of Curricula, Methods and Practices: The Case of English Language Teaching in Morocco”. Mr. Soudani attends the Ph.D. study at university in Marrakech (Morocco).

We are very pleased to introduce him and welcome him in our research group.